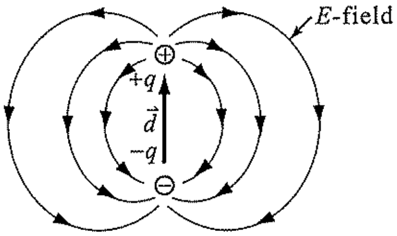
範題 26

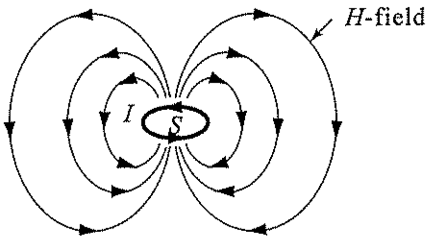
Sketch the electric field lines of an elecinc dipoie and the magnetic flux lines of a magnetic dipole respectively. 【96清大醫工】

【解答】

(1) Electric dipole $\vec{P} = q\vec{d}$



(2) Magnetic dipole $\vec{m} = IS\hat{a}_S$



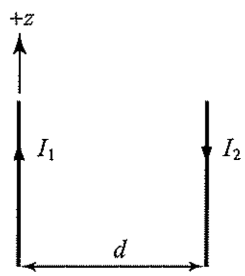
範題 27

Determine the force per unit length between two infinitely long parallel conducting wires carrying currents I_1 and I_2 in the opposite direction. The wires are separated by a distance d.

【96清大醫工】

$I_1 \to \vec{B}_{12} \to \vec{F}_{12} = I_2 d\vec{\ell} \times \vec{B}_{12}$

【解答】



由安培定律 $\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I_{in}$

$2\pi rB = \mu_0 I_1$

$\Rightarrow \vec{B}_{12} = \dfrac{\mu_0 I_1}{2\pi d}\hat{a}_\phi$

$$\vec{F}_{12}=I_2 d\vec{\ell}\times\vec{B}_{12}=I_2(-\hat{a}_z)\times\frac{\mu_0 I_1}{2\pi d}\hat{a}_\phi$$

$$=\frac{\mu_0 I_1 I_2}{2\pi d}\hat{a}_r$$（排斥力）

範題 28

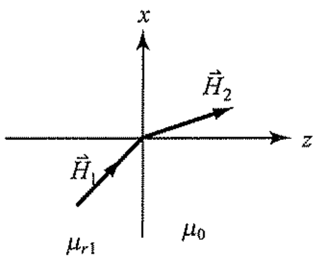
An interface between two magnetic media lies in the $x-y$ plane at $z=0$. Above the $x-y$ plane ($z>0$), there exists a magnetic material with $\mu_{r1}=3.0$ and a field $\vec{H}_1=4.0\hat{a}_x+5.0\hat{a}_z$ (A/m). Below the $x-y$ plane ($z<0$) is free space.

(1) Assuming the boundary is free of surface current, find $\vec{H}_2$ and the angle that $\vec{H}_2$ makes with a normal to the surface?

(2) Assuming the boundary has a surface current $\vec{J}=6.0\hat{a}_x$ (A/m), Find $\vec{H}_2$. 【96台科大光電】

$\vec{B}_{1n}=\vec{B}_{2n}$，$\hat{a}_{n2}\times(\vec{H}_1-\vec{H}_2)=\vec{J}_S$

【解答】



(1) $\vec{H}_1=4\hat{a}_x+5\hat{a}_z$

$\vec{B}_1=\mu_{r1}\vec{H}=12\hat{a}_x+15\hat{a}_z$

由於邊界上無表面電流：

$$\vec{B}_{1n}=\vec{B}_{2n}=15\hat{a}_z\Rightarrow\vec{H}_{2n}=\frac{\vec{B}_{2n}}{\mu_0}=15\hat{a}_z\cdots\cdots ①$$

$$\vec{H}_{1t}=\vec{H}_{2t}=4\hat{a}_x\cdots\cdots ②$$

由①②可知：

$\vec{H}_2=4\hat{a}_x+15\hat{a}_z$

且與法線之 $\theta_t=\tan^{-1}\dfrac{4}{15}$

(2) 由(1)可知 $\vec{H}_{2n}=15\hat{a}_z$

且 $\hat{a}_{n2}\times(\vec{H}_1-\vec{H}_2)=\vec{J}_S$

$$-\hat{a}_z\times[(4\hat{a}_x+5\hat{a}_z)-(H_{2x}\hat{a}_x+H_{2y}\hat{a}_y+15\hat{a}_z)]=6\hat{a}_x$$

$$\hat{a}_y(-4+H_{2x})+\hat{a}_x(-H_{2y})=6\hat{a}_x$$

$\therefore H_{2x} = 4,\quad H_{2y} = -6$

$\therefore \vec{H}_2 = 4\hat{a}_x - 6\hat{a}_y + 15\hat{a}_z$

範題 29

A transmission line in air with permeability μ_0 consists of two long parallel conducting wires of radius b. The axes of the wires are separated by a distance D, which is much larger than b. If the two wires carry currents of magnitude I in opposite directions, find in detail the expressions for:

(1) the total self-inductance per unit length of the transmission line.

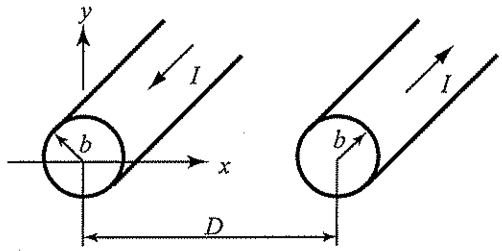
(2) the force per unit length between the two parallel wires.

【96台科大電子】

(1) $I \to \vec{B} \to \phi \to L = \dfrac{\phi}{I}$ ， $L = L_{內} + L_{外}$

(2) $\vec{F}_{12} = I_2 d\vec{\ell} \times \vec{B}_{12}$

【解答】



(1)先求 $L_{外}$ ：

①安培定律：(柱座標)

$$\oint_C \vec{B}\cdot d\vec{\ell} = \mu_0 I$$

$$2\pi r B = \mu_0 I$$

$$\Rightarrow \vec{B} = \frac{\mu_0 I}{2\pi r}\hat{a}_\phi$$

② $\phi = \int_S \vec{B}\cdot d\vec{S}$ (取單位長度電感)

$$\Rightarrow \phi = \int_b^{D-b} \frac{\mu_0 I}{2\pi r} \cdot 1 \cdot dr = \frac{\mu_0 I}{2\pi} \ell\text{n} \frac{D-b}{b}$$

由於存在左右導線，因此總磁通量 ϕ_{total}

$$\phi_{total} = \phi_{左} + \phi_{右} = \frac{\mu_0 I}{\pi} \ell\text{n} \frac{D-b}{b}$$

③ $L = \dfrac{\phi_{total}}{I} = \dfrac{\mu_0}{\pi} \ell\text{n} \dfrac{D-b}{b}$

求 $L_{內}$：如同coaxiel line之解法

單一導線之 $L_{內} = \dfrac{\mu_0}{8\pi}$

存在左右兩導線 $\Rightarrow L_{total} = L_{內} + L_{外} = \dfrac{\mu_0}{4\pi}$

$$\therefore L = L_{內} + L_{外} = \frac{\mu_0}{\pi} \ell\text{n} \frac{D-b}{b} + \frac{\mu_0}{4\pi}$$

(2) $\vec{F}_{12} = I_2 d\vec{\ell} \times \vec{B}_{12} = I(-\hat{a}_z) \times \dfrac{\mu_0 I}{2\pi D} \hat{a}_\phi = \dfrac{\mu_0 I^2}{2\pi D} \hat{a}_r$（排斥力）

範題30

Current flows with density $\vec{J} = J_0\left(\dfrac{r}{a}\right)\hat{a}_z$ A/m^2 along an infinitely long solid cylindrical wire of radius a having the z-axis as its axis. Find $\vec{H}$ everywhere and plot H_ϕ versus $\vec{r}$. 【96成大電機】

【解答】

利用安培定律 $\oint_C \vec{H} \cdot d\vec{\ell} = I_{in}$

(1) $r \le a$ 區：

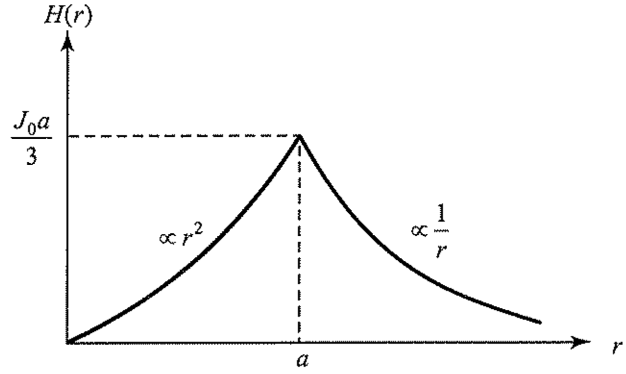
$$H \cdot 2\pi r = \int \vec{J} \cdot d\vec{S} = \int_0^{2\pi} \int_0^r \frac{J_0 r}{a} \cdot r d\phi dr = \frac{2\pi J_0 r^3}{3a}$$

$$\Rightarrow \vec{H} = \frac{J_0 r^2}{3a} \hat{a}_\phi$$

(2) $r \ge a$ 區：

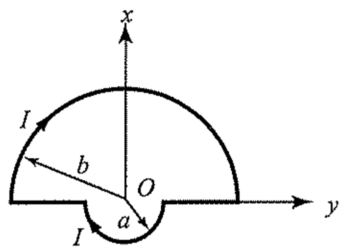
$$H \cdot 2\pi r = \int \vec{J} \cdot d\vec{S} = \int_0^{2\pi} \int_0^a \frac{J_0 r}{a} \cdot r d\phi dr = \frac{2\pi J_0 a^2}{3}$$

$$\Rightarrow \vec{H} = \frac{J_0 a^2}{3r}\hat{a}_\phi$$



範題 31

Two semicircle loops of radius a and b have a common center and their ends are joined by straight wires, as shown in Fig.. If the two loops are perpendicular to each other (xy plane and yz plane), what is the $\vec{B}$ field at the center?



【96成大物理】

利用重疊定律。

【解答】

$$\vec{B} = \vec{B}_{上半} + \vec{B}_{下半}$$

而於 y 方向之電流，並不會貢獻磁場於 O 點。

已知一線圈電流貢獻於原點之磁場爲 $\vec{B}$

$$\vec{B} = \frac{\mu_0 I}{2r}\hat{a}_z$$

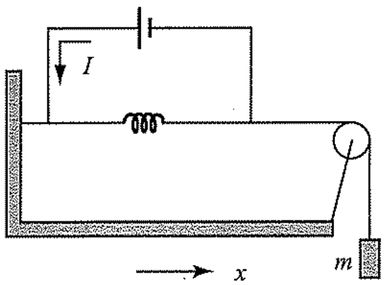
而半圈之 $\vec{B}$ 則爲

$$\vec{B} = \frac{\mu_0 I}{4r}\hat{a}_z$$

$$\therefore \vec{B} = \vec{B}_{上半} + \vec{B}_{下半} = \left(\frac{\mu_0 I}{4a} + \frac{\mu_0 I}{4b}\right)\hat{a}_z$$

範題 32

A circular coil of length L and radius a has N turns of wire and carries a current I. A block with mass m is connected to the coil by a light thread that passes over a frictionless pulley, as shown in Fig.. What is the current if the coil is not stretched. 【96成大物理】



固定電流迴路 $\vec{F}_I$ 系統，$\vec{F}_I = \nabla W_m$。

【解答】

利用虛位移法求 ∇L

由安培定律 $\oint_C \vec{H} \cdot d\vec{\ell} = I_{in}$

$\Rightarrow H = nI$　n 爲單位長度之線圈匝數，$n = \dfrac{N}{L}$

若線圈受重力而右移 x，則 $n = \dfrac{N}{L+x}$

$$W_m = \frac{1}{2}\int_V \mu_0 H^2 dV = \frac{1}{2}\mu_0 \cdot \left(\frac{N}{L+x}\right)^2 I^2 \cdot \pi a^2 \cdot (L+x) = \frac{\mu_0 N^2 I^2 \pi a^2}{2(L+x)}$$

$$\vec{F}_I = \nabla W_m = \frac{2W_m}{2x}\hat{a}_x = -\frac{\mu_0 N^2 I^2 \pi a^2}{2(L+x)^2}\hat{a}_x$$

$$\left|\vec{F}_I\right|_{x=0} = \frac{\mu_0 N^2 I^2 \pi a^2}{2L^2} = mg = \left|\vec{F}_{重}\right|$$

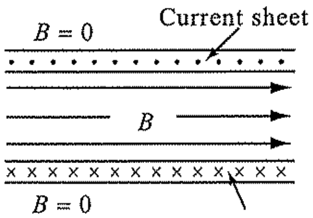
$$\Rightarrow I = \frac{L}{Na}\sqrt{\frac{2mg}{\mu_0 \pi}}$$

範題 33

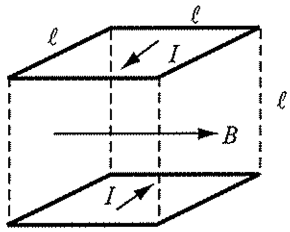
Consider two infinite conducting parallel planes carrying equal and opposite currents, as shown in Fig. (a), produce a uniform field inside and zero field outside. Image a cubical section, as shown in the three-dimensional view of Fig. (b), is cut from the two parallel sheets. A structure such as this is called field cell if its dimensions are small.

(1)Find the inductance of the cell.

(2)Use the result in part (1), discuss the meaning of permeability μ_0.



(a) Current sheet



(b)【96成大物理】

【解答】

(1)安培定律：$\oint_C \vec{B}\cdot d\vec{\ell}=\mu I$

$\ell\cdot B=\mu_0 I$

$B=\mu_0\dfrac{I}{\ell}$

$\phi=\int_S \vec{B}\cdot d\vec{S}=\dfrac{\mu_0 I}{\ell}\cdot\ell^2=\mu_0 I\ell$

$L=\dfrac{\phi}{I}=\mu_0\ell$

(2)由(1)之結果可知，μ_0爲單位長度之電感。

範題34

The quantity $\dfrac{B^2}{\mu_0}$ has units of:

(A)J (B)$\dfrac{\text{J}}{\text{H}}$ (C)$\dfrac{\text{J}}{\text{m}}$ (D)$\dfrac{\text{J}}{\text{m}^3}$ (E)$\dfrac{\text{H}}{\text{m}^3}$. 【96成大光電】

【解答】

$\dfrac{B^2}{\mu_0}=\mu_0 H^2$爲磁能之能量密度

即單位爲J/m^3