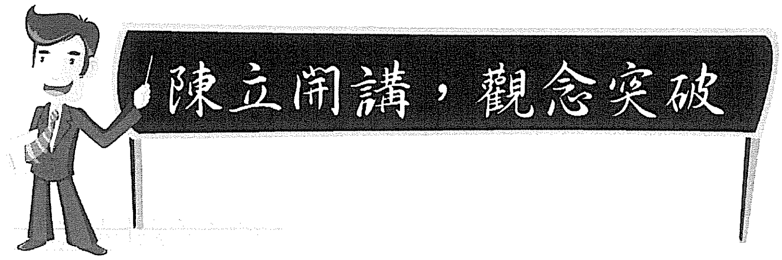

積分因子公式的證明：

(1) 若 $\boxed{I = e^{\int f(x)dx}}$ 為 <u>O.D.E.</u> $M(x,y)dx + N(x,y)dy = 0$ 之積分因子

則 $e^{\int f(x)dx}M(x,y)dx + e^{\int f(x)dx}N(x,y)dy = 0$ 為正合，

必滿足『**交換微分會相等**』的口訣

$$\frac{\partial\{e^{\int f(x)dx}M(x,y)\}}{\partial y} = \frac{\partial\{e^{\int f(x)dx}N(x,y)\}}{\partial x}$$

$$e^{\int f(x)dx}\frac{\partial M}{\partial y} = f(x)e^{\int f(x)dx}N(x,y) + e^{\int f(x)dx}\frac{\partial N}{\partial x}$$

將上式同除以 $e^{\int f(x)dx}$，得 $\frac{\partial M}{\partial y} = f(x)N + \frac{\partial N}{\partial x}$ ⇨ $\boxed{\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(x)}$

(2) 若 $\boxed{I = e^{-\int f(y)dy}}$ 為 <u>O.D.E.</u> $M(x,y)dx + N(x,y)dy = 0$ 之積分因子

則 $e^{-\int f(y)dy}M(x,y)dx + e^{-\int f(y)dy}N(x,y)dy = 0$ 為正合，

必滿足『**交換微分會相等**』的口訣

$$\frac{\partial\{e^{-\int f(y)dy}M(x,y)\}}{\partial y} = \frac{\partial\{e^{-\int f(y)dy}N(x,y)\}}{\partial x}$$

$$-f(y)e^{-\int f(y)dy}M(x,y)+e^{-\int f(y)dy}\frac{\partial M}{\partial y}=e^{-\int f(y)dy}\frac{\partial N}{\partial x}$$

將上式同除以 $e^{-\int f(y)dy}$，得 $-f(y)M(x,y)+\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$

⇨ $\boxed{\dfrac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{M}=f(y)}$

(3) 若 $\boxed{I=e^{-\int f(x+y)d(x+y)}}$ 為 <u>O.D.E.</u> $M(x,y)dx+N(x,y)dy=0$ 積分因子

則 $e^{-\int f(x+y)d(x+y)}M(x,y)dx+e^{-\int f(x+y)d(x+y)}N(x,y)dy=0$ 為正合，

必滿足『**交換微分會相等**』的口訣

$$\frac{\partial\{e^{-\int f(x+y)d(x+y)}M(x,y)\}}{\partial y}=\frac{\partial\{e^{-\int f(x+y)d(x+y)}N(x,y)\}}{\partial x}$$

$$-f(x+y)e^{-\int f(x+y)d(x+y)}M(x,y)+e^{-\int f(x+y)d(x+y)}\frac{\partial M}{\partial y}$$

$$=-f(x+y)e^{-\int f(x+y)d(x+y)}N(x,y)+e^{-\int f(x+y)d(x+y)}\frac{\partial N}{\partial x}$$

將上式同除以 $e^{-\int f(x+y)d(x+y)}$，得

$$-f(x+y)M(x,y)+\frac{\partial M}{\partial y}=-f(x+y)N(x,y)+\frac{\partial N}{\partial x}$$

⇨ $\boxed{\dfrac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{M-N}=f(x+y)}$

(4) 若 $\boxed{I=e^{-\int f(xy)d(xy)}}$ 為 <u>O.D.E.</u> $M(x,y)dx+N(x,y)dy=0$ 積分因子

則 $e^{-\int f(xy)d(xy)}M(x,y)dx + e^{-\int f(xy)d(xy)}N(x,y)dy = 0$ 為正合，

必滿足『**交換微分會相等**』的口訣

$$\frac{\partial\{e^{-\int f(xy)d(xy)}M(x,y)\}}{\partial y} = \frac{\partial\{e^{-\int f(xy)d(xy)}N(x,y)\}}{\partial x}$$

$$-xf(xy)e^{-\int f(xy)d(xy)}M(x,y) + e^{-\int f(xy)d(xy)}\frac{\partial M}{\partial y}$$

$$= -yf(xy)e^{-\int f(xy)d(xy)}N(x,y) + e^{-\int f(xy)d(xy)}\frac{\partial N}{\partial x}$$

將上式同除以 $e^{-\int f(xy)d(xy)}$，得 $-xf(xy)M(x,y) + \frac{\partial M}{\partial y} = -yf(xy)N(x,y) + \frac{\partial N}{\partial x}$

⇨ $$\boxed{\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{xM - yN} = f(xy)}$$