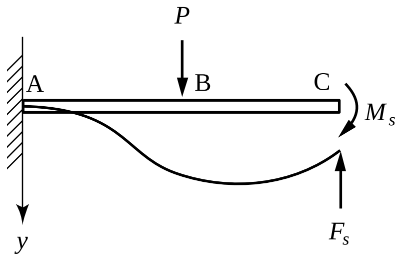
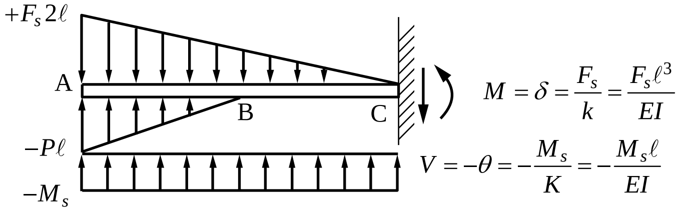
⑷共軛樑法




$\uparrow +,\quad \sum F_y = 0$

$$\Rightarrow \frac{1}{EI}\left[-\frac{1}{2}\times 2\ell\times 2F_s\ell+\frac{1}{2}\times\ell\times P\ell+2\ell\times M_s\right]-\left(-\frac{M_s\ell}{EI}\right)=0$$

$$\Rightarrow 2F_s\ell^2-\frac{1}{2}P\ell^2-3M_s\ell=0 \quad \cdots\cdots ①$$

$\curvearrowleft +,\quad \sum M_{右}=0$

$$\Rightarrow \frac{1}{EI}\left[+\frac{1}{2}\times 2\ell\times 2F_s\ell\times\left(\frac{2}{3}\times 2\ell\right)-\frac{1}{2}\times\ell\times P\ell\times\left(\ell+\frac{2}{3}\ell\right)-2\ell\times M_s\times\ell\right]$$

$$+\frac{F_s\ell^3}{EI}=0$$

$$\Rightarrow +\frac{11}{3}F_s\ell^3-\frac{5}{6}P\ell^3-2M_s\ell^2=0 \quad \cdots\cdots ②$$

聯立式①～②可解得

$$F_s=+\frac{3}{14}P,\quad M_s=-\frac{1}{42}P\ell$$

同理，由靜力平衡條件即可求得

$$R=+\frac{11}{14}P,\quad M=-\frac{23}{42}P\ell$$

⑸卡氏定理

$$U=\int_0^{\ell}\frac{[F_s\cdot x-M_s]^2}{2EI}dx+\int_{\ell}^{2\ell}\frac{[F_sx-M_s-P(x-\ell)]^2}{2EI}dx$$

$$\frac{\partial U}{\partial F_s}=-\delta=-\frac{F_s\ell^3}{EI}$$

$$=\int_0^{\ell}\frac{[F_sx-M_s]\cdot x}{EI}dx+\int_{\ell}^{2\ell}\frac{[F_sx-M_s-P(x-\ell)]\cdot x}{EI}dx$$

$$\Rightarrow -F_s\ell^3=\frac{8}{3}F_s\ell^3-2M_s\ell^2-\frac{5}{6}P\ell^3 \quad\cdots\cdots\cdots\text{①}$$

$$\frac{\partial U}{\partial M_s}=-\theta=-\frac{M_s\ell}{EI}$$

$$=\int_0^{\ell}\frac{[F_sx-M_s]\cdot(-1)}{EI}dx+\int_{\ell}^{2\ell}\frac{[F_sx-M_s-P(x-\ell)]\cdot(-1)}{EI}dx$$

$$\Rightarrow -M_s\ell=-2F_s\ell^2+2M_s\ell+\frac{1}{2}P\ell^2 \quad\cdots\cdots\cdots\text{②}$$

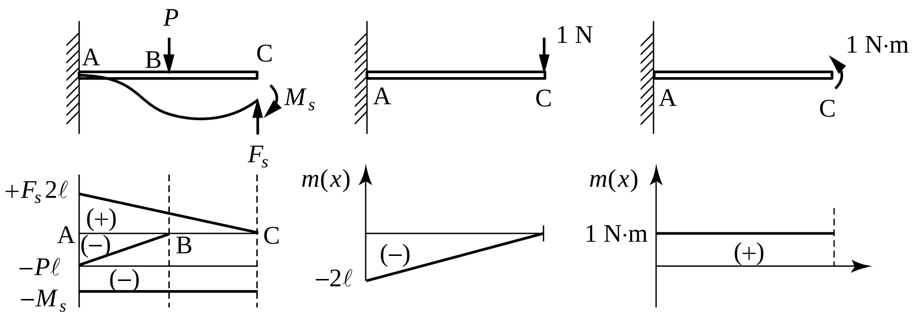
聯立式①～②，可解得

$$F_s=\frac{3}{14}P,\quad M_s=-\frac{1}{42}P\ell$$

再利用靜力平衡條件即可解得

$$R=+\frac{11}{14}P,\quad M=-\frac{23}{42}P\ell$$

⑹單位荷重法




$$\delta=\frac{F_s\ell^3}{EI}=\frac{1}{EI}\Big[\frac{1}{2}\times2\ell\times2F_s\ell\times\frac{2}{3}\times(-2\ell)-\frac{1}{2}\times\ell\times P\ell\times\frac{5}{6}\times(-2\ell)$$

$$-M_s \times 2\ell \times \frac{1}{2} \times (-2\ell)\Big]$$

$$\Rightarrow F_s\ell^3 = -\frac{8}{3}F_s\ell^3 + \frac{5}{6}P\ell^3 + 2M_s\ell^2 \cdots\cdots ①$$

$$\theta = \frac{M_s\ell}{EI} = \frac{1}{EI}\left[\frac{1}{2}\times 2\ell \times 2F_s\ell \times 1 - \frac{1}{2}\times \ell \times P\ell \times 1 - M_s \times 2\ell \times 1\right]$$

$$\Rightarrow M_s\ell = 2F_s\ell^2 - \frac{1}{2}P\ell^2 - 2M_s\ell \cdots\cdots ②$$

聯立式①～②，可解得

$$F_s = +\frac{3}{14}P, \quad M_s = -\frac{1}{42}P\ell$$

最後，由靜力平衡條件即可解得

$$R = +\frac{11}{14}P, \quad M = -\frac{23}{42}P\ell$$

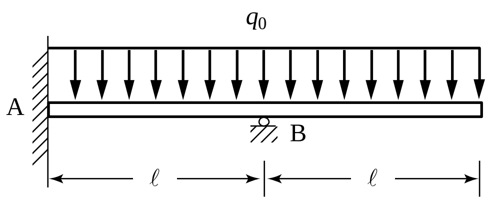
## 12-2 靜不定樑問題

本節對前節所介紹之前六種不同解法，各以數例說明各解法之解題觀念，最後並各列舉幾題較難之靜不定問題以加強應用能力，分述如下：

*1.* 奇函數法

**範題 2**

如圖示之懸臂樑，試求簡支點B之反力為何？




➠應用奇函數法時，積分二次會出現2個積分常數，而未知反力有3個，故必須再利用2個靜力平衡方程式及3個邊界條件才可得解。而由 A 為固