題型8-9 非極小相位系統之波德圖

範題 12

Hand-sketch the bode plot for the following system:

(1) $\dfrac{10s+10000}{s^3+3s^2+102s+100}$

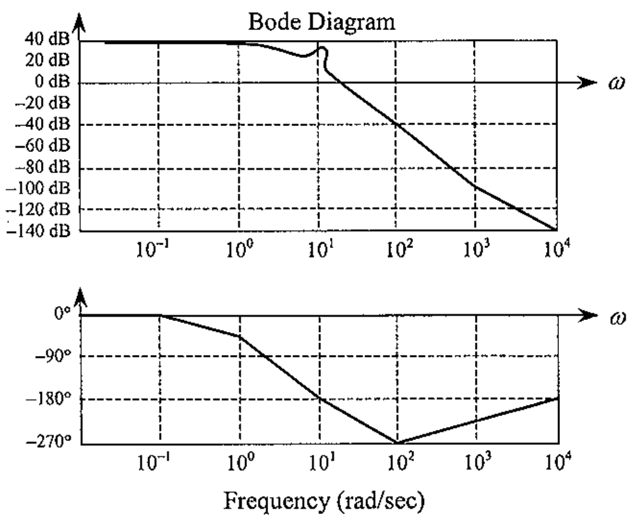
(2) $\dfrac{-10s+10000}{s^3+3s^2+102s+100}$ （96交大機械）

【解析】

(1)寫為繪波德圖之標準形式如下

$$\frac{10s+10000}{s^3+3s^2+102s+100}=\frac{100\left(1+\dfrac{1}{1000}s\right)}{(1+s)\left(1+\dfrac{2}{10}s+\dfrac{s^2}{10^2}\right)}$$

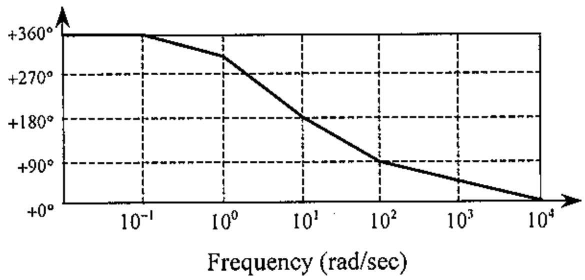
直接應用斜率變化關係，即可繪出波德圖如下



(2)寫為繪波德圖之標準型式如下

$$\frac{-10s+10000}{s^3+3s^2+102s+100}=\frac{(-200)\times\left(-1+\frac{1}{1000}s\right)}{(1+s)\left(1+\frac{2}{10}s+\frac{s^2}{10^2}\right)}$$

此為極小相位轉移函數，其波德圖之幅量曲線與(1)小題相同，而相位曲線如下



範題 13

Bode plots

(1)In Figure 1, derive the transfer function from r_v to v for the dashed block, with $K_v=3$.

(2)Sketch the Bode diagram including Gain (in dB), Phase (in rad), and Frequencies (in Hz, $\omega=2\pi f$). Indicate the asymptotic curves/slopes, DC gain/phase, and the corner (-3 dB) frequency.

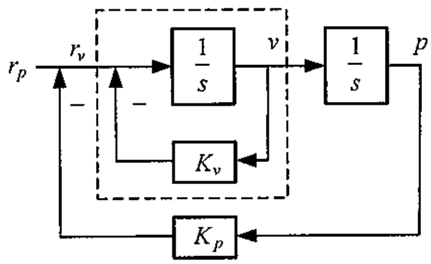


Figure 1

(3)In Figure 1, derive the transfer function from r_p to p, with $K_v=3$ and $K_p=2$.

(4)Sketch the Bode diagram including Gain (in dB), Phase (in rad), and Frequencies (in Hz, $\omega=2\pi f$). Indicate the asymptotic curves/slopes, DC gain/phase, and the corner (-3 dB) frequency.

題型11-14 轉移函數存在極零點對消與可控制性及可觀測性之關係

範題 18

Given the following system $\begin{cases} \dot{x}_1 = -ax_1 - bx_2 \\ \dot{x}_2 = -cx_2 + dx_1 + u \\ y = x_1 \end{cases}$,

where u is the input, y is the output, x_1, x_2 are state variables, and a, b, c, d are constants. If the system is both controllable and observable, what are the conditions on a, b, c, d, and what is the transfer function? What is the physical meaning of the state variable? （96台大機械）

【解析】

(1) $A = \begin{bmatrix} -a & -b \\ -c & d \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [1 \quad 0]$

所以轉移函數$G(s)$為

$$G(s) = C(sI - A)^{-1}B = \frac{-b}{s^2 + (a-d)s - (ad+bc)}$$

(2)若轉移函數不存在極點零點可對消，則系統必為可控制性及可觀測性，由$G(s)$中可看出不存在極點與零點可對消，因此只要$b \neq 0$則系統即為可控制性且為可觀測性。

(3)依定義知狀態變數為描述系統內部物理狀態之變數，亦即狀態變數代表系統內部之動態行為，而一般以轉移函數所描述之系統只能知道其輸入與輸出之關係，並不能知道系統內部之動態，若欲掌握或控制系統內部動態，則採用狀態方程式表示法。

範題 19

The block diagram of a control system is shown in Fig. P5.

(1)Determine the transfer function $\dfrac{C(s)}{R(s)}$.

(2)Determine what value (or values) of K must be avoided if the system is to be both completely state controllable and observable.

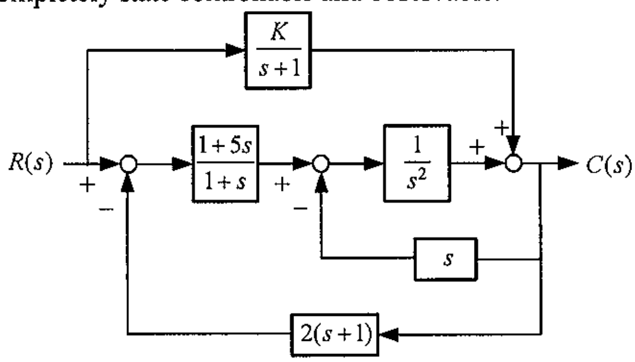


Figure P5 （95台科大電機）

【解析】

(1)前向路徑

$$P_1=\frac{1+5s}{1+s}\times\frac{1}{s^2}=\frac{(5s+1)}{(s+1)s^2}$$

$$P_2=\frac{K}{s+1}$$

迴路

$$L_1=-\frac{1}{s^2}\times s=-\frac{1}{s}$$

$$L_2=-\frac{1+5s}{1+s}\times\frac{1}{s^2}\times 2(s+1)$$

$$=-\frac{2(5s+1)}{s^2}$$

直接應用梅森增益公式求轉移函數如下

$$\frac{C(s)}{R(s)}=\frac{\frac{5s+1}{(s+1)s^2}\times 1+\frac{K}{s+1}\times 1}{1-\left(-\frac{1}{s}-\frac{2(5s+1)}{s^2}\right)}=\frac{Ks^2+5s+1}{(s+1)(s^2+11s+2)}$$

(2)閉迴路系統之極點為 $s=-1,-0.1849,-10.8151$，若系統要可控且可觀，則轉移函數不能存在相同的極點與零點。令

$$Ks^2+5s+1=0 \Rightarrow K=\frac{-(5s+1)}{s^2}$$

分別將 $s=-1,-0.1849$ 及 -10.8151 代入，可求得 $K=4,-2.2084,$ 0.4538，亦即若 $K=4,-2.2084$ 或0.4538時，轉移函數將存在極點與零點可對消，此時系統之可控制性與可觀測性必不可能同時都成立。

範題 20

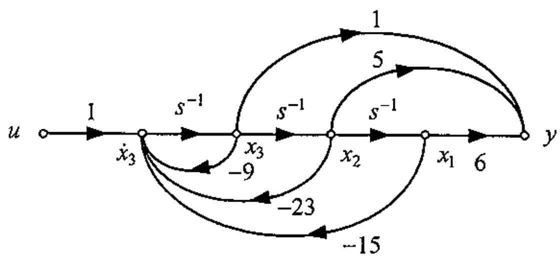
(1) If $G(s)=\dfrac{s^2+5s+6}{s^3+9s^2+23s+15}$, find the controllable canonical form: $\begin{cases}\dot{x}=Ax+bu\\ y=cx\end{cases}$.

(2)Is the system in the above observable? Why?

（94中央電機、89交大電機與控制）

【解析】

(1)採用直接分解法可繪出狀態圖如下



狀態方程式為