「統計學歷屆試題詳解Ⅰ」增補資料

（109.12.08）

【P109-64】

【清華大學科管所（甲組）】

四、A bank with a branch located in a commercial district of a city has the business objective of developing an improved process for serving customers during the noon-to-1 p.m. lunch period. Management decides to first study the waiting time in the current process. The waiting time is defined as the number of minutes that elapses from when the customer enters the line until he or she reaches the teller window. Data are collected from a random sample of 5 customers. These data are:

4.21　5.55　3.02　5.13　2.34

Suppose that another branch, located in a residential area, is also concerned with improving the process of serving customers in the noon-to-1 p.m. lunch period. Data are collected from a random sample of 4 customers. These data are:

9.66　5.90　8.02　8.73

㈠At the 0.05 level of significance, is there evidence of a difference in the variability of waiting time between the two branches?（10%）

㈡On the basis of the results in ㈠, which t-test should you use to compare the means of waiting time between the two branches? At the 0.05 level of significance, is there evidence of a difference in the mean waiting time between the two branches?（10%）

解：思考點：假設檢定。

4.21 5.55 3.02 5.13 2.34　　$\overline{X}=4.05$，$S^2=1.86$

9.66 5.9 8.02 8.73　　$\overline{X}=8.08$，$S^2=2.56$

㈠ $\alpha=0.05$

$$\begin{cases} H_0：\sigma_1^2=\sigma_2^2 \\ H_1：\sigma_1^2\neq\sigma_2^2 \end{cases}$$

$$RR=\{F \mid F>\underset{\substack{\parallel \\ 15.1}}{F_{0.025}(4.3)}，\varphi<\underset{\substack{\parallel \\ \frac{1}{9.98}=0.1}}{F_{0.975}(4.3)}\}$$

$\because F=\dfrac{1.86}{2.56}=0.7266\notin RR$ $\therefore$無證據支持 $\sigma_1^2 \neq \sigma_2^2$

(二) H_0 ： $\mu_1=\mu_2$

H_1 ： $\mu_1 \neq \mu_2$

$$t=\frac{\bar{x}_1-\bar{x}_2-(\mu_1-\mu_2)}{\sqrt{S_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \sim t(n_1+n_2-2)$$

$$|t|^*=\left|\frac{4.05-8.08}{\sqrt{2.16\left(\frac{1}{5}+\frac{1}{4}\right)}}\right|=4.09\in RR \qquad S_p^2=\frac{1.86\times 4+2.56\times 3}{5+4-2}=2.16$$

$RR=\{t \mid |t|>t_{0.025}(7)(=2.365)\}$

有證據顯示 $\mu_1 \neq \mu_2$

五、The following table shows the joint distribution of two discrete random variable, X and Y.

	$Y=4$	$Y=6$	$Y=8$
$X=1$	0.2	0	0.2
$X=2$	0	0.2	0
$X=3$	0.2	0	0.2

(一)Compute $E(X\,|\,Y)$. （10%）

(二)Compute the correlation between X and Y. （10%）

(三)Are X and Y independent? （5%）

解：思考點：間斷型隨機變數。

(一) $E\left(\frac{X}{Y}\right)=\left(\frac{1}{4}+\frac{1}{8}+\frac{2}{6}+\frac{3}{4}+\frac{3}{8}\right)\times 0.2=\frac{11}{30}$

(二) $\rho\times Y=\frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$

$\because Cov(X,Y)=E(XY)-E(X)E(Y)=12-2\times 6=0$

其中 $E(X)=2$ ， $E(Y)=6$ ， $Var(X)=0.8$ ， $Var(Y)=3.2$

$E(XY)=(4+12+12+8+24)\times 0.2=12$

$\therefore \rho_{XY}=0$

(三)No.

$\because \underset{0}{\underset{\parallel}{P(X=2,Y=4)}}\neq \underset{0.2}{\underset{\parallel}{P(X=2)}}\,\underset{0.4}{\underset{\parallel}{P(Y=4)}}=0.08$

【P109-81】

【成功大學企管所（乙組）】

五、A production line operation is tested for filling weight accuracy using the following hypotheses $H_0: \mu = 16$ vs $H_a: \mu \neq 16$ where μ denotes the population mean of the filling weight. The sample size is 16 and the population standard deviation is 1. Use Type Ⅰ error rate $\alpha = 0.05$ to make decision. Recall that the 95 and 97.5 percentiles of the standard normal are 1.645 and 1.96.

㈠What would a Type Ⅱ error mean in this situation? （5%）

㈡If the true filling weight is 15.51, what is the Type Ⅱ error rate for this problem? （5%）

解：思考點：假設檢定。

㈠Type Ⅱ error：H_1 爲真之下，錯誤接受 H_0

即 $\mu \neq 16$，但卻認定 $\mu = 16$

㈡ $\begin{cases} H_0: \mu = 16 \\ H_a: \mu \neq 16 \end{cases}$，$\alpha = 0.05$

$n = 16$，$\sigma = 1$

依題意已知此假設檢定的拒絕域如下：

$$R = \left\{ \bar{X} \mid \bar{X} < 16 - 1.96 \times \frac{1}{\sqrt{16}} \ or\ \bar{X} > 16 + 1.96 \times \frac{1}{\sqrt{16}} \right\}$$

$$= \{ \bar{X} \mid \bar{X} < 15.51 \ or\ \bar{X} > 16.49 \}$$

$$\therefore \beta = P(15.51 \le \bar{X} \le 16.49 \mid \mu = 15.51)$$

$$= P\left(\frac{15.51 - 15.51}{1/\sqrt{16}} \le z \le \frac{16.49 - 15.51}{1/\sqrt{16}} \right)$$

$$= P(0 \le z \le 3.92) = 0.5$$

【P109-56】

【交通大學科管所】

一、From all the possible subsets of a set of 10 elements, two subsets A and B are selected randomly and independently. Find $P(A \subset B)$. （20%）

解：Let $X = (a_1, a_2, a_3, \ldots, a_{10})$, for each $a_i \in X$ $(i = 1, 2, \ldots, 10)$

We have four cases: ① $a_i \in A$ and $a_i \in B$ ② $a_i \in A$ and $a_i \notin B$

③ $a_i \notin A$ and $a_i \in B$ ④ $a_i \notin A$ and $a_i \notin B$

針對$1 \le i \le 10$，令C_i爲一事件滿足$A \cap \{i\} \le B \cap \{i\}$。而滿足此情況有上述Cases①、②及④，故$P(C_i) = \dfrac{3}{4}$，所以對所有$i \ne j$時，$C_i$與$C_j$爲互相獨立，則：

$$P(A \le B) = P(C_1 \cap C_2 \cap \cdots \cap C_{10}) = \prod_{k=1}^{10} P(C_k) = \left(\frac{3}{4}\right)^{10}$$