「統計學歷屆試題詳解Ⅱ」增補資料

（109.12.08）

【P109-72】

【台北大學金融合經所】

一、Suppose X_1, X_2 are independent and both follow continuous uniform distribution $U(0,1)$. Please find the probability $\Pr(X_1^2+X_2^2<1)$. （10%）

解：思考點：連續機率分配應用。

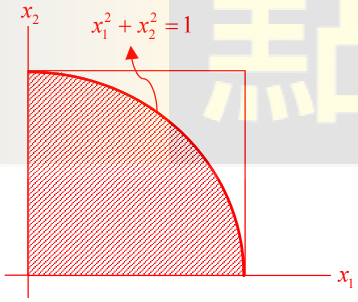
$$X_1,X_2 \overset{\text{iid}}{\sim} u(0,1)$$

$$\Rightarrow f(x_1,x_2)=\begin{cases}1 & ，0\le x_1\le 1，0\le x_2\le 1\\ 0 & ，0/w\end{cases}$$

$$\therefore P(X_1^2+X_2^2<1)=\int_0^1\int_0^{\sqrt{1-x_1^2}} 1dx_2dx_1 \underset{\uparrow}{=} \int_0^{\frac{\pi}{2}}\int_0^1 rdrd\theta$$

$$令\begin{cases}x_1=r\cos\theta\\ x_2=r\sin\theta\end{cases}\Rightarrow J=\begin{vmatrix}\cos\theta & -r\sin\theta\\ r\sin\theta & r\cos\theta\end{vmatrix}=r$$

$$=\int_0^{\frac{\pi}{2}}\frac{1}{2}d\theta=\frac{1}{2}\times\frac{\pi}{2}=\frac{\pi}{4}$$



四、Let $Y_1, Y_2, \ldots, Y_{11}$ be a random sample of size $n=11$ from Group 1 with mean μ_1 and variance σ_1^2; and their sample mean is $\bar{\mu}_1$ and sample variance is $S_1^2 = \dfrac{\sum_{i=1}^{11}(Y_i - \bar{\mu}_1)}{(n-1)}$. Let $Y_{12}, Y_{13}, \ldots, Y_{24}$ be the other random sample of size $m=13$ from Group 2 with population mean μ_2 and variance σ_2^2; and their sample mean is $\bar{\mu}_2$ and sample variance is $S_2^2 = \dfrac{\sum_{i=12}^{24}(Y_i - \bar{\mu}_2)}{(m-1)}$. Let D_i be a dummy, whose value is 1 if Y_i from Group 1, and is 0 if Y_i from Group 2, $i = 1, 2, \ldots, 24$. A simple regression is estimated as follows:

$$\hat{Y}_i = 1.66 - 0.63 D_i,$$

whose $SSE = \sum_{i=1}^{24}(Y_i - \hat{Y}_i) = 6.6$.

㈠What is the value of $(\bar{\mu}_2 - \bar{\mu}_1)$? （5%）

㈡Assume $\sigma_1^2 = \sigma_2^2$, please find an estimate of $Var(\bar{\mu}_2 - \bar{\mu}_1)$. (Hint: $SSE = (n-1)S_1^2 + (m-1)S_2^2$) （10%）

解：思考點：Dummy Variables（二元變數）。

$\{Y_{1i}\}_{i=1}^{11} \sim (\mu_1, \sigma_1^2)$

$\{Y_{2i}\}_{i=12}^{24} \sim (\mu_2, \sigma_2^2)$

given $SSE = 6.6$，$\hat{Y}_i = 1.66 - 0.63 D_i$（which is dummy）

㈠令 $\hat{Y}_i = \hat{\alpha} + \hat{\beta} z_i + \varepsilon_i$，$i = 1, \ldots, 11, 12, \ldots, 24$

其中 $z_i = \begin{cases} 1 & \text{，若} i = 1, \ldots, 11 \text{ (group 1)} \\ 0 & \text{，其他} \end{cases}$

透過最小平方法可得：

$$\begin{cases} -\bar{\mu}_1 + \hat{\alpha} + \hat{\beta} - \bar{\mu}_2 + \hat{\alpha} = 0 \\ \bar{\mu}_1 = \hat{\alpha} + \hat{\beta} \end{cases} \Rightarrow \begin{cases} \hat{\alpha} = \hat{\mu}_2 \\ \hat{\beta} = \bar{\mu}_1 - \bar{\mu}_2 \end{cases}$$

$\therefore (\bar{\mu}_2 - \bar{\mu}_1) = -\hat{\beta}$

$\bar{\mu}_2 - \bar{\mu}_1 = 0.63$

(二)Assume $\sigma_1^2=\sigma_2^2$，$Var(\bar{\mu}_2-\bar{\mu}_1)=Var(-\hat{\beta})=Var(\hat{\beta})$

$$\because \widehat{Var}(\hat{\beta})=\frac{\hat{\sigma}^2}{S_{zz}}\underset{\uparrow}{=}MSE\times\frac{1}{S_{zz}}$$

$$\hat{\sigma}^2=MSE$$

上式中：

$$S_{zz}=\sum_{i=1}^{24}Z_i-\frac{\left(\sum Z_i\right)^2}{24}$$

$$=11-\frac{11^2}{24}=\frac{143}{24}\left(=\frac{1}{\left(\frac{24}{143}\right)}=\frac{1}{\left(\frac{1}{11}+\frac{1}{13}\right)}\right)$$

又 $MSE=\frac{SSE}{n+m-2}=\frac{6.6}{11+13-2}$ $(=S_p^2)$

$$\therefore \widehat{Var}(\hat{\beta})=S_p^2\left(\frac{1}{n}+\frac{1}{m}\right)=\frac{6\cdot 6}{11+13-2}\times\left(\frac{1}{11}+\frac{1}{13}\right)=\frac{36}{715}$$

更正原試題【P109-147】

【中山大學經濟所】

一、Suppose that $f(x)=3x^2$ for $0<x<1$ and $f(y|x)=\frac{2y}{x^2}$ for $0<y<x$.

Find $f(x|y)$. （20%）

二、Let Y be uniformly distributed on the interval (0,1). Conditional on $Y=y$, let X be uniformly distributed on the interval $(0,y)$. Find $E(X)$ and $Var(X)$. （20%）

三、Let $X_1,\ldots,X_n$ be a random sample from

$$f(x;\theta)=(\theta+1)x^{\theta},\quad 0<x<1.$$

Find the method of moments estimator for θ. （20%）

四、A random sample of size $n=1$ is drawn from a uniform pdf density over the interval $[0,\theta]$. We decide to test

$$H_0: \theta=2,$$

versus

$$H_1: \theta\neq 2$$

by rejecting H_0 if either $x\le 0.1$ or $x\ge 1.9$, where x is the value drawn. Find α. Also, find β if the true value of θ is 2.5. （20%）

五、Let

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \sim N\left(\begin{bmatrix} 170 \\ 68 \\ 10 \end{bmatrix}, \begin{bmatrix} 400 & 64 & 128 \\ 64 & 16 & 0 \\ 128 & 0 & 256 \end{bmatrix}\right).$$

Find $f(X_1|X_2=75, X_3=36)$. （20%）