$$3.1795+2.776\sqrt{0.3116}\sqrt{\frac{1}{6}+\frac{(2-5.5)^2}{45.5}}\Bigg)$$

$\Rightarrow (2.1564, 4.2026)$

難易度：🏆🏆🏆🏆

例題 1.20

將符合迴歸分析的自變數 X 與因變數 Y 共12對輸入電腦，得 X 之平均數 $\bar{X}=34$，原點在(0,0)之迴歸方程式 $\hat{Y}=343.699+3.221X$，以及迴歸分析之ANOVA表如下：

變異來源	平方和	自由度
迴歸	17032	1
殘差	25220	10

⑴檢定迴歸模型是否與橫軸（ X 軸）平行？（取 $\alpha=0.05$ ）

⑵預測 $E[Y\mid X=30]$ 之95%信賴區間。

⑶求條件變異數 $\sigma^2_{Y|X}$ 的信賴區間。（台大商研、東吳企管）

➠⑴① $H_0:\beta_1=0$

② $H_1:\beta_1\neq 0$

③ $\alpha=0.05$

④危險域 $C=\{F\mid F>F_{0.05}(1,10)=4.9646\}$

⑤計算： $F=\dfrac{MSR}{MSE}=\dfrac{17032/1}{25220/10}=6.7534\in C$

⑥結論：拒絕 H_0，亦即表示斜率係數不為0，故與 X 軸不平行。

⑵ $\because \hat{Y}_{X=30}=343.699+3.221\times 30=440.329$

又 $SSR=\hat{\beta}_1^2\sum_{i=1}^{n}(X_i-\bar{X})^2=17032$，故知

$$\sum_{i=1}^{n}(X_i-\bar{X})^2=\frac{SSR}{\hat{\beta}_1^2}=1641.664$$

故 $E[Y\mid X=30]$ 之95%信賴區間為

$$\Bigg(\hat{Y}_X-t_{0.025}(10)\sqrt{MSE}\sqrt{\frac{1}{n}+\frac{(X-\bar{X})^2}{S_{XX}}},$$

$$\hat{Y}_X + t_{0.025}(10)\sqrt{MSE}\sqrt{\frac{1}{n}+\frac{(X-\bar{X})^2}{S_{XX}}}\Bigg)$$

$$\Rightarrow\left(440.329-2.228\sqrt{2522}\sqrt{\frac{1}{12}+\frac{(30-34)^2}{1641.664}},\right.$$

$$\left.440.329+2.228\sqrt{2522}\sqrt{\frac{1}{12}+\frac{(30-34)^2}{1641.664}}\right)$$

$$\Rightarrow(406.193,474.465)$$

(3) $\sigma^2_{Y|X}$ 之95%信賴區間為

$$\Rightarrow\left(\frac{SSE}{\chi^2_{0.025}(10)},\frac{SSE}{\chi^2_{0.975}(10)}\right)\Rightarrow\left(\frac{25220}{20.4831},\frac{25220}{3.24697}\right)\Rightarrow(1231.259,7767.241)$$

難易度：🏆🏆🏆🏆

例題 1.21

Using the following statistics, answer problems (1) to (3).

$\sum X=137$ ，$\sum Y=253$ ，$n=25$ ，$\sum XY=1609$ ，

$\sum X^2=895$ ，$\sum Y^2=2943$

(1)A least squares line was derived, what is the slope of this line?

(A)2.11 (B)1.54 (C)3.01 (D)1.66 (E)2.83.

(2)A least squares line was derived, what is the intercept of this line?

(A)2.11 (B)1.54 (C)3.01 (D)1.66 (E)2.83.

(3)A 95% confidence interval for the mean value of Y at $X=6.0$ was derived (say, from A to B). What is the value of B?

(A)13.57 (B)10.16 (C)12.05 (D)11.48 (E)14.14. （成大財金）

➠(1)(B)；

$$\hat{\beta}_1=\frac{n\sum XY-\left(\sum X\right)\left(\sum Y\right)}{n\sum X^2-\left(\sum X\right)^2}=\frac{25\times1609-(137\times253)}{25\times895-(137)^2}=\frac{5564}{3606}\doteqdot1.543$$

(2)(D)；

$$\hat{\beta}_0=\bar{Y}-\hat{\beta}_1\bar{X}=\left(\frac{253}{25}\right)-1.543\times\left(\frac{137}{25}\right)\doteqdot1.664$$

(3)(D)；

$$MSE=\frac{\sum_{i=1}^{n}Y_i^2-\hat{\beta}_0\sum_{i=1}^{n}Y_i-\hat{\beta}_1\sum_{i=1}^{n}X_iY_i}{n-2}$$

$$=\frac{2943-(1.664\times 253)-(1.543\times 1609)}{23}\doteqdot 1.7096$$

又 $S_{XX}=\sum_{i=1}^{n}(X_i-\bar{X})^2=895-\frac{(137)^2}{25}=144.24$

且 $X=6$ 時，$\hat{Y}_{X=6}=1.664+1.543\times 6=10.922$

$\therefore \mu_{Y|X=6}$ 之95%信賴區間為

$$\Rightarrow\left(\hat{Y}_X-t_{\frac{\alpha}{2}}(n-2)\sqrt{MSE}\cdot\sqrt{\frac{1}{n}+\frac{(X-\bar{X})^2}{S_{XX}}},\ \hat{Y}_X+t_{\frac{\alpha}{2}}(n-2)\sqrt{MSE}\cdot\sqrt{\frac{1}{n}+\frac{(X-\bar{X})^2}{S_{XX}}}\right)$$

$$\Rightarrow\left(10.922-2.069\sqrt{1.7096}\sqrt{\frac{1}{25}+\frac{(6-5.48)^2}{144.24}},\ 10.922+2.069\sqrt{1.7096}\sqrt{\frac{1}{25}+\frac{(6-5.48)^2}{144.24}}\right)$$

$$\Rightarrow(10.3684,11.4756)$$

難易度：🏆🏆

例題 1.22

In a study to determine how the skill in doing a complex assembly job is influenced by the amount of training, 15 new recruits were given varying amounts of training ranging between 3 and 12 hours. After the training, their times to perform the job were recorded. After denoting x = duration of training (in hours) and y = time to do the job (in minutes), the following summary statistics were calculated

$\bar{x}=7.2$，$\bar{y}=45.6$，$\sum_{i=1}^{n}(x_i-\bar{x}_i)^2=33.6$，

$\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})=-57.2$，$\sum_{i=1}^{n}(y_i-\bar{y}_i)^2=160.2$

⑴Determine the equation of the best fitting straight line.

⑵Do the data substantiate the claim that the job time decreases with more hours of training? Use $\alpha = 0.05$.

⑶Estimate the mean job time for 9 hours of training and construct a 95% confidence interval.

⑷Find the predicted *y* for $x = 35$ hours and comment on the result.

（淡江企管）

➠⑴ $\hat{\beta}_1 = \dfrac{S_{XY}}{S_{XX}} = \dfrac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^{n}(x_i - \overline{x})^2} = \dfrac{-57.2}{33.6} \fallingdotseq -1.7024$

$\hat{\beta}_0 = \overline{Y} - \hat{\beta}_1 \overline{X} = 45.6 + 1.7024 \times 7.2 \fallingdotseq 57.8573$

故最佳直線方程式為 $\hat{Y} = 57.8573 - 1.7024X$

⑵① $H_0 : \beta_1 \geq 0$

② $H_1 : \beta_1 < 0$

③ $\alpha = 0.05$

④ $C = \{t \mid t < -t_{0.05}(13) = -1.771\}$

⑤計算：因 $SSE = S_{YY} - \hat{\beta}_1^2 S_{XX}$

$= 160.2 - (-1.7024)^2 \cdot (33.6)$

$= 62.82163$

故 $MSE = \dfrac{SSE}{n-2}$

$= \dfrac{62.82163}{15-2} = 4.83243$

又 $t = \dfrac{\hat{\beta}_1}{\sqrt{\dfrac{MSE}{S_{XX}}}} = \dfrac{-1.7024}{\sqrt{\dfrac{4.83243}{33.6}}} = -4.489 \in C$

⑥結論：拒絕 H_0，即 β_1 顯著小於0。

⑶當 $X = 9$ 時，Y 之估計值為

$\hat{Y}_{X=9} = 57.8573 - 1.7024 \times 9 = 42.5357$

故 $\mu_{Y|X=9}$ 之95%信賴區間為

$$\left(\hat{Y}_{X=9} \mp t_{0.025}(13)\sqrt{MSE}\sqrt{\frac{1}{n}+\frac{(X-\bar{X})^2}{S_{XX}}}\right)$$

$$\Rightarrow \left(42.5357 \mp 2.16 \cdot \sqrt{4.83243}\sqrt{\frac{1}{15}+\frac{(9-7.2)^2}{33.6}}\right)$$

$$\Rightarrow (40.6181, 44.4533)$$

(4) $\hat{Y}_{X=35} = 57.8573 - 1.7024 \times 7.2 = 45.6$

因 $X \in [3,12]$，故知 $X = 35$ 超出範圍很多，故此預測值並不合理。

難易度：🏆🏆🏆🏆

例題 1.23

Based on the following 10 pairs of data,

X	6	3	4	–3	2	–4	0	–1	–5	–2
Y	10	7	8	1	6	2	5	3	2	1

and assuming the following linear regression model,

$Y = \alpha + \beta X + U$,

where U has zero mean and variance equal to σ^2.

answer the following questions.

(1) Compute the least-squares point estimates of α and β, and the coefficient of determination.

(2) Calculate variance estimates for $\hat{\alpha}$ and $\hat{\beta}$, and the covariance estimate between $\hat{\alpha}$ and $\hat{\beta}$.

(3) With 10% significance level, test the hypothesis: $H_0 : \sigma^2 = 1$ using the following critical values:

$Z_{0.90} = 1.28$, $Z_{0.95} = 1.65$; $t_{0.90,8} = 1.40$, $t_{0.95,8} = 1.86$;

$t_{0.90,9} = 1.38$, $t_{0.95,9} = 1.83$; $t_{0.90,10} = 1.37$, $t_{0.95,10} = 1.81$;

$\chi^2_{0.9,8} = 13.36$, $\chi^2_{0.95,8} = 15.51$; $\chi^2_{0.9,9} = 14.68$, $\chi^2_{0.95,9} = 16.92$;

$\chi^2_{0.9,10} = 15.99$, $\chi^2_{0.95,10} = 18.31$.　（政大國貿）

➠(1) $\because \sum_{i=1}^{n} X_i = 0$，$\sum_{i=1}^{n} X_i^2 = 120$，$\sum_{i=1}^{n} Y_i = 45$，$\sum_{i=1}^{n} Y_i^2 = 293$，$\sum_{i=1}^{n} X_i Y_i = 99$