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多重選擇題, 每題 5 分;每題答案可能不只一個。

1. Given two random variables, x and y with finite second moments, which of following statement(s) about independence is correct?
   (A) If x and y are independent with each other, then they are uncorrelated.
   (B) If x and y are uncorrelated with each other, then they are definitely independent of each other.
   (C) If $P(x=ay=b)=P(x=a)$ then x and y are independent of each other.
   (D) If $E(x|y)$ is a constant, then x and y are independent of each other.

2. A random variable $x \sim N^{+}(0,\sigma^2)$, where N^{+} is a half-normal distribution that x is always positive and has a pdf, then we know:
   (A) $f(x)=\frac{1}{\sqrt{2\pi\sigma}}\exp\left(-\frac{x^2}{2\sigma^2}\right), \quad -\infty<x<\infty.$
   (B) $f(x)=\frac{\sqrt{2}}{\sigma\sqrt{\pi}}\exp\left(-\frac{x^2}{2\sigma^2}\right), \quad 0<x<\infty.$
   (C) $E[x]=\sqrt{\frac{\sigma^2}{\pi}}.$
   (D) $Var(x)>\sigma^2.$

3. Which of following statement(s) about the Central Limit Theorem (CLT) is correct?
   (A) If $\text{plim}x=\mu_x$, then CLT is held.
   (B) If $N>30$, then $\bar{x}\overset{A}{\sim}N(\mu_x,\sigma^2)$ under the conditions that the random variable x has finite mean and variance.
   (C) If $N>30$, then $\bar{x}\overset{A}{\sim}N\left(\mu_x,\frac{\sigma^2}{30}\right)$ under the conditions that the random variable x has finite mean and variance.
   (D) $\bar{x}$ does not converge to normal distribution if x is a random walk process $x_t=x_{t-1}+w_t, \quad w_t\sim N(0,1).$

4. Given cdf of a random variable x: $F_X(a) = \frac{a^2}{36}$, then we have...

(A) The pdf $f_X(a) = \frac{a}{18}$, $0 \le a \le 6$.

(B) $E(x) = 4$.

(C) $E(x^2) = 2$.

(D) $Var(x) = 2$.

5. Let $u = (x-b)^2$. x is a random variable and $E\left[(x-b)^2\right]$ exists. Which of following statement(s) is correct?

(A) $E(u)$ is minimal when $b = 0$.

(B) When $b = 0$, u is the variance of x.

(C) $E(u)$ is minimal when $b = E(x)$.

(D) When $b = E(x)$, u is the variance of x.

6. A random sample $\{x_1, x_2, \ldots, x_N\}$ is sampled, where $x_i \overset{i.d.}{\sim} N(\mu_x, \sigma_i^2)$, which means x is independent distributed to a normal distribution (note that heteroskedasticity exists). A point estimator is calculated as$\tilde{x} = \sum_{i=1}^{N} a_i x_i$; $\sum_{i=1}^{N} a_i = 1$. Then which of the following statement(s) is correct?

(A) $\tilde{x}$ is unbiased to μ_x.

(B) $\tilde{x}$ is a best linear unbiased estimator of μ_x.

(C) $\tilde{x}$ is a best unbiased estimator of μ_x.

(D) $\tilde{x}$ is a consistent estimator of μ_x, if $a_i = 1/N$.

7. A random sample $\{x_1, x_2, \ldots, x_N\}$ is sampled, where $x \overset{i.i.d.}{\sim} N(\mu_x, \sigma^2)$. We define a downside standard deviation of x as $\hat{\sigma}_x^d = \sqrt{\frac{1}{N}\sum_{i=1}^{N} \max(0, -x_i)^2}$ and $\hat{\sigma}_x = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu_x)^2}$ is the classical standard deviation. Then we will obtain:
   (A) $\hat{\sigma}_x^d > \hat{\sigma}_x$.
   (B) $\hat{\sigma}_x^d < \hat{\sigma}_x$.
   (C) $\hat{\sigma}_x^d$ becomes larger when x is more positive skewed.
   (D) Both of $\hat{\sigma}_x^d$ and $\hat{\sigma}_x$ are biased estimators of population standard deviation σ.

8. A random variable $x \sim (\mu_x, 1)$, according to Chebyshev inequality, the lower bound of $P(\{|x - \mu_x| < 2\})$ is
   (A) 0
   (B) 0.75
   (C) 0.95
   (D) 0.99

9. Two random variables x and y have following relations: $y = b_0 + b_1 x + u$, and $x = a_0 + a_1 y + v$. Error terms u and v are independent of each other, which both obey standardized normal distribution. We can know...
   (A) OLS estimator $\hat{b}_1$ is a consistent estimator.
   (B) If $b_1 > 0$ and $a_1 > 0$, then OLS estimator $\hat{b}_1$ is downward inconsistent.
   (C) If $b_1 > 0$ and $a_1 < 0$, then OLS estimator $\hat{b}_1$ is downward inconsistent.
   (D) If $b_1 > 1$ and $a_1 > 0$, then OLS estimator $\hat{b}_1$ is upward inconsistent.

10. Using following OLS estimations (see table below) for regression model $Y_i = b_0 + b_1X_i + b_2D_i + b_3X_iD_i + u_i$, in which X is a continuous variable, and D is a binary variable, please answer which of the following statement(s) is correct:

Summary statistics			
R-sq	0.90	Mean of Y	1
Adj. R-sq	0.89	Mean of X	0
N	50	Mean of D	0.6
		Mean of X*D	0.02

	Coef	SD	t-stat
Intercept	1.14	0.21	5.51
X	1.80	0.27	6.74
D	-0.24	0.27	-0.91
X*D	1.44	0.32	4.53

ANOVA

	DF	SS	MS	F
Regression	3	345.61	115.20	135.53
Residual	46	39.10	0.85	
Sum	49	384.71		

(A) Average marginal effect for a unit increase in X is 1.8.

(B) Average marginal effect for a unit increase in X is 2.66.

(C) The mean squared error of $Y_i = b_0 + b_1X_i + b_2D_i + b_3X_iD_i + u_i$ is smaller than a simple linear regression model: $Y_i = b_0 + b_1X_i + v_i$.

(D) $\widehat{Cov(X, D)} \approx 0.02$.

11. Let $\{(X_i, Y_i)\}_{i=1}^q$ be a sequence of independently and $N(0, I_2)$-distributed random vectors. Define the random variable:

$$Z_i(q) = \frac{X_i}{\sqrt{\sum_{i=1}^q Y_i^2/q}}.$$

Which of the following is right?

(A) $\mathbb{E}[Z_i(q)] = 0$.

(B) $\mathbb{E}[Z_i^3(q)] = 0.$

(C) $\mathbb{E}[Z_i^4(q)] = 3$, as $q \to \infty$.

(D) $\mathbb{E}[Z_i^6(q)] = 15$, as $q \to \infty$.

12. Let $(Y_1, Y_2, \ldots, Y_n)'$ be a random vector with the distribution $N(0, \Sigma)$ and the covariance matrix:

$$\Sigma = \begin{bmatrix} 1 & \rho & \rho^2 & \ldots & \rho^{n-1} \\ \rho & 1 & \rho & \ldots & \rho^{n-2} \\ \rho^2 & \rho & 1 & \ldots & \rho^{n-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{n-1} & \rho^{n-2} & \rho^{n-3} & \ldots & 1 \end{bmatrix}$$

for some $\rho > 0$ and $n \geq 3$. Define the sample average: $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$. Which of the following is right?

(A) $\mathrm{var}[\bar{Y}] = \frac{1}{n} + 2\sum_{i=1}^{n}\left(1 - \frac{i}{n}\right)\rho^i$.

(B) $\lim_{n\to\infty} \mathrm{var}[\bar{Y}] = 0$.

(C) $\mathrm{var}[n^{1/2}\bar{Y}] < 1 + 2\sum_{i=1}^{n}\left(1 - \frac{i}{n}\right)\rho^i$.

(D) $\lim_{n\to\infty} \mathrm{var}[n^{1/2}\bar{Y}] = 1$, if $\rho = n^{-1/2}$.

13. Let X be a $\chi^2(k)$-distributed random variable, and Y be a $N(0,1)$-distributed random variable. Suppose that X and Y are independent. Define the random variable: $W = X^{1/2}Y$. Which of the following is right?

(A) $\mathbb{E}[W^4] = 15$, if $k = 1$.

(B) $\mathbb{E}[W^4] = 30$, if $k = 2$.

(C) $\mathbb{E}[W^4] = 45$, if $k = 3$.

(D) None of the above choices (A)-(C).

14. Let $\{(X_i, Z_i)'\}_{i=1}^n$ be a sequence of independently and $N(0, \Sigma)$-distributed random vectors, with the covariance matrix:

$$\Sigma = \begin{bmatrix} 1 & \rho \\ \rho & 2 \end{bmatrix},$$

for some constant $\rho > 0$. Define the random variable: $Y_i = X_i^2 + Z_i$, for all i's. Consider a linear regression: $Y_i = \beta_0 + \beta_1 X_i + e_i$, where (β_0, β_1) is a parameter vector, and e_i is an error term, for all i's. Let $(\hat{\beta}_0, \hat{\beta}_1)$ be the ordinary least squares estimator of (β_0, β_1). Which of the following estimators is consistent for ρ, as $n \to \infty$?

(A) $\hat{\rho} = \hat{\beta}_1$

(B) $\hat{\rho} = \hat{\beta}_1 + 29(\hat{\beta}_0 - 1)$

(C) $\hat{\rho} = (\hat{\beta}_1 - \hat{\beta}_0)^2$

(D) None of the above choices (A)-(C).

15. Let $\{X_i\}_{i=1}^n$ be a sequence of independently and $N(1, 2)$-distributed random variables. Define the sample average:

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i.$$

Which of the following statistics has the limiting distribution $\chi^2(1)$, as $n \to \infty$?

(A) $\frac{n}{2}\left(\bar{X}^2 - 2\bar{X} + 1\right)$.

(B) $\frac{n}{8}\left(\bar{X}^4 - 2\bar{X}^2 + 1\right)$.

(C) $\frac{n}{16}\left(\bar{X}^6 - 2\bar{X}^3 + 1\right)$.

(D) $\frac{n}{32}\left(\bar{X}^8 - 2\bar{X}^4 + 1\right)$.

16. Let $\{(Y_i, X_{1i}, X_{2i})'\}_{i=1}^{n}$ be a sequence of independently and $N(0, \Sigma)$-distributed random vectors, where Σ is a 3×3 covariance matrix. Consider the following two regressions:

$$Y_i = \beta_0 + \beta_1 X_{1i} + e_{1i},$$

and

$$Y_i = b_1 X_{1i} + b_2 X_{2i} + e_{2i},$$

for all i's, with the parameters: $\beta_0, \beta_1, b_1, b_2$ and the error terms e_{1i} and e_{2i}. Let $(\hat{\beta}_0, \hat{\beta}_1)$ and $(\hat{b}_1, \hat{b}_2)$ be the ordinary least squares estimators of (β_0, β_1) and (b_1, b_2), respectively. Also, define the following two coefficients of determination:

$$R_1^2 = \frac{\sum_{i=1}^{n}(Y_{1i} - \bar{\hat{Y}}_1)^2}{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}$$

and

$$R_2^2 = \frac{\sum_{i=1}^{n}(Y_{2i} - \bar{\hat{Y}}_2)^2}{\sum_{i=1}^{n}(Y_i - \bar{Y})^2},$$

where $\bar{Y} = n^{-1}\sum_{i=1}^{n} Y_i$, $Y_{1i} = \hat{\beta}_0 + \hat{\beta}_1 X_{1i}$, $Y_{2i} = \hat{b}_1 X_{1i} + \hat{b}_2 X_{2i}$, $\bar{\hat{Y}}_1 = n^{-1}\sum_{i=1}^{n} Y_{1i}$, $\bar{\hat{Y}}_2 = n^{-1}\sum_{i=1}^{n} Y_{2i}$. Denote $\hat{e}_{1i} := Y_i - \hat{Y}_{1i}$ and $\hat{e}_{2i} := Y_i - \hat{Y}_{2i}$. Which of the following is right?

(A) $R_1^2 = 1 - \frac{\sum_{i=1}^{n} \hat{e}_{1i}^2}{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}$

(B) $R_2^2 = 1 - \frac{\sum_{i=1}^{n} \hat{e}_{2i}^2}{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}$

(C) $R_1^2 \neq 1 - \frac{\sum_{i=1}^{n} \hat{e}_{1i}^2}{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}$

(D) $R_2^2 \neq 1 - \frac{\sum_{i=1}^{n} \hat{e}_{2i}^2}{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}$

17. Let $(X, Y, Z)'$ be a random vector with the distribution $N(0, I_3)$. According to the Cauchy-Schwarz inequality, which of the following results is right?

(A) $\mathbb{E}[XY] \leq 1$.

(B) $\mathbb{E}[XY^2] \leq \sqrt{3}$.

(C) $\mathbb{E}[X^2Z^2] \leq 3$.

(D) $\mathbb{E}[X^2Y^3Z^3] \leq 15\sqrt{3}$.

18. Let $\{(W_i, X_i, Y_i, Z_i)'\}_{i=1}^n$ be a sequence of random vectors that satisfies the following properties:

$$W_i = X_i^2 + Y_i^2 + Z_i^2,$$

$$\begin{bmatrix} Y_i \\ Z_i \end{bmatrix} \Bigg| X_i \sim N\left(\begin{bmatrix} X_i \\ X_i^2 \end{bmatrix}, \begin{bmatrix} X_i^2 & X_i^3 \\ X_i^3 & X_i^4 \end{bmatrix} \right)$$

and X_i is $N(0, 1)$-distributed, for all i's. Consider a linear regression:

$$W_i = \alpha_0 + \alpha_1 X_i + \alpha_2 X_i^2 + \alpha_3 X_i^3 + \alpha_4 X_i^4 + e_i,$$

where $(\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4)$ is a parameter vector, and e_i is an error term, for all i's. Also, let $(\hat{\alpha}_0, \hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3, \hat{\alpha}_4)$ be the ordinary least squares (OLS) estimator of $(\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4)$. Which of the following is the probability limit of $(\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4)$, as $n \to \infty$?

(A) $(0, 0, 2, 0, 4)$

(B) $(0, 2, 0, 3, 4)$

(C) $(0, 0, 3, 0, 2)$

(D) None of the above choices (A)-(C).

19. Let $\{X_i\}_{i=1}^n$ be a sequence of independently and $U(0,1)$-distributed random variables. Define the statistic:

$$F_n(x) = \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}(X_i \leq x),$$

for some fixed $x \in (0,1)$, where $\mathbb{1}(A)$ is the indicator function which equals one if A is true (otherwise, zero). Which of the following is right?

(A) The limiting distribution of $n^{1/2}(F_n(x)/x - 1)$ is $N(0, 1/(x-1))$, as $n \to \infty$.

(B) The probability limit of $2F_n(x) - 1$ is $2x - 1$, as $n \to \infty$.

(C) The limiting variance of $n^{1/2}(F_n(x)/x^2 - 1/x)$ is $1/x^3 - 1/x^2$, as $n \to \infty$.

(D) The probability limit of $F_n(x)(1 - F_n(x))$ is the same as the limiting variance of $n^{1/2}(F_n(x) - x)$, as $n \to \infty$.

20. Let $\{(X_i, e_i)\}_{i=1}^n$ be a sequence of independently and identically distributed random vectors with the properties: $X_i \sim N(0,1)$ and

$$e_i \mid X_i \sim N(0, X_i^2).$$

Define the random variable:

$$Y_i = \beta X_i + e_i,$$

where β is a constant, for all i's. Which of the following is right?

(A) The statistic $\dfrac{\sum_{i=1}^n X_i Y_i}{\sum_{i=1}^n X_i^2}$ is consistent for β, as $n \to \infty$.

(B) The statistic $\dfrac{\sum_{i=1}^n X_i^2 Y_i}{\sum_{i=1}^n X_i^3}$ is inconsistent for β, as $n \to \infty$.

(C) The statistic $\dfrac{\sum_{i=1}^n |X_i| Y_i}{\sum_{i=1}^n |X_i| X_i}$ is inconsistent for β, as $n \to \infty$.

(D) The statistic $\dfrac{\sum_{i=1}^n X_i Y_i |X_i|^{-2}}{\sum_{i=1}^n (X_i |X_i|^{-1})^2}$ is not less efficient than $\dfrac{\sum_{i=1}^n X_i Y_i}{\sum_{i=1}^n X_i^2}$, as $n \to \infty$.