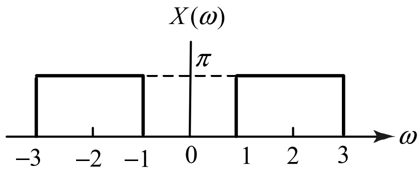
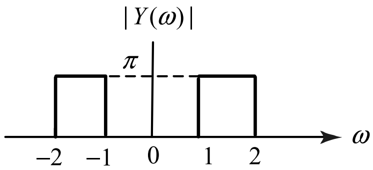
(1)



(2) $H(\omega)=\begin{cases} e^{-j\omega}, & |\omega|<2 \\ 0, & \text{otherwise} \end{cases}$

$|H(\omega)|=\begin{cases} 1, & |\omega|<2 \\ 0, & \text{otherwise} \end{cases}$ ， $\angle H(\omega)=\begin{cases} -\omega, & |\omega|<2 \\ 0, & \text{otherwise} \end{cases}$

$|Y(\omega)|=|X(\omega)||H(\omega)|$



(3) $Y(\omega)=[\pi p_1(\omega-1.5)+\pi p_1(\omega+1.5)]e^{-j\omega}$

$$\pi p_1(\omega-1.5)+\pi p_1(\omega+1.5)=\frac{1}{2\pi}2\pi p_1(\omega)*[\pi\delta(\omega-1.5)+\pi\delta(\omega+1.5)]$$

而 $2\pi p_1(\omega)\xrightarrow{\mathfrak{F}^{-1}}1\cdot \text{sinc}(t/2\pi)$

$$\pi\delta(\omega-1.5)+\pi\delta(\omega+1.5)\xrightarrow{\mathfrak{F}^{-1}}\cos(1.5t)$$

$$\therefore \pi p_1(\omega-1.5)+\pi p_1(\omega+1.5)\xrightarrow{\mathfrak{F}^{-1}}\text{sinc}\left(\frac{t}{2\pi}\right)\cos(1.5t)$$

$$\Rightarrow [\pi p_1(\omega-1.5)+\pi p_1(\omega+1.5)]e^{-j\omega}\xrightarrow{\mathfrak{F}^{-1}}\text{sinc}\left(\frac{t-1}{2\pi}\right)\cos[1.5(t-1)]=y(t)$$

試題29

Two signals $x(t)$ and $y(t)$ are said to be orthogonal if $\int_{-\infty}^{\infty}x(t)y^*(t)dt=0$, where $y^*(t)$ denotes the complex-conjugate of $y(t)$. Suppose two band-pass signals are $x(t)=p(t)\cdot\ \exp(j2\pi nt)$ and $y(t)=p(t)\cdot\ \exp(j2\pi kt)$, respectively, where n and k are different integers and $p(t)$ is a baseband pulse with Fourier transform

$$P(f)=\begin{cases}1, \text{if} \left|f\right| \le 0.5 \\ 0, \text{if} \left|f\right| > 0.5\end{cases}$$

(1)Find and plot the pulse $p(t)$.

(2)Find and plot the Fourier transforms of $x(t)$ and $y(t)$, respectively.

(3)Prove the following property: $\int_{-\infty}^{\infty} x(t)y^*(t)dt = \int_{-\infty}^{\infty} X(f)Y^*(f)df$, where $X(f)$ and $Y(f)$ are the Fourier transforms of $x(t)$ and $y(t)$, respectively.

(4)Is $x(t)$ and $y(t)$ orthogonal? Prove it.

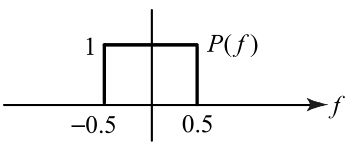
(5)Suppose the channel outputs for $x(t)$ and $y(t)$ are $u(t)=x(t)*h(t)$ and $v(t)=y(t)*h(t)$, respectively, where $h(t)$ is the impulse response of the linear-time invariant channel and * denotes convolution. Is $u(t)$ and $v(t)$ orthogonal for any $h(t)$? Prove it.

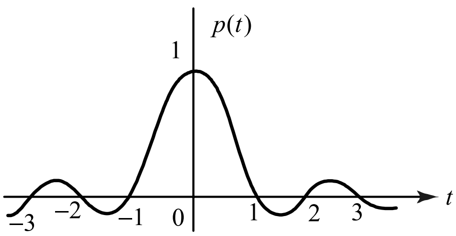
（93交大電資學院專班電信組，訊號與系統）

➠(1) $P(f)=\begin{cases}1, & \text{if} \left|f\right| \le 0.5 \\ 0, & \text{if} \left|f\right| > 0.5\end{cases} = \text{rect}(f)$

$\therefore p(t)=\mathfrak{F}^{-1}\{P(f)\}=\int_{-\infty}^{\infty} \text{rect}(f)\cdot e^{j2\pi ft}df$

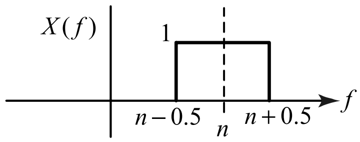
$=\text{sinc}(t)=\dfrac{\sin(\pi t)}{\pi t}$





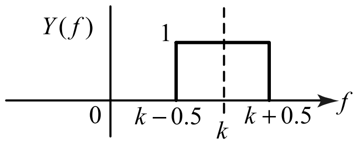
(2) $X(f)=\mathfrak{F}\{x(t)\}=\mathfrak{F}\{p(t)\cdot \exp(j2\pi nt)\}=P(f-n)$

（此為$n>0$之情況）



$Y(f)=\mathfrak{F}\{y(t)\}=\mathfrak{F}\{p(t)\cdot\exp(j2\pi kt)\}=P(f-k)$

（此為 $k>0$ 之情況）



(3) $\int_{-\infty}^{\infty}x(t)y^*(t)dt=\int_{-\infty}^{\infty}x(t)\left[\int_{-\infty}^{\infty}Y(f)e^{j2\pi ft}df\right]^*dt$

$$=\int_{-\infty}^{\infty}x(t)\cdot\int_{-\infty}^{\infty}Y^*(f)e^{-j2\pi ft}df\ dt$$

$$=\int_{-\infty}^{\infty}Y^*(f)\int_{-\infty}^{\infty}x(t)e^{-j2\pi ft}dt\ df$$

$$=\int_{-\infty}^{\infty}X(f)Y^*(f)df$$ 得證

(4) n 與 k 為不同的整數，因此

$$\int_{-\infty}^{\infty}x(t)y^*(t)dt=\int_{-\infty}^{\infty}X(f)Y^*(f)df=\int_{-\infty}^{\infty}P(f-n)P^*(f-k)df$$

$$=\int_{-\infty}^{\infty}0\cdot df=0$$

故 $x(t)$ 與 $y(t)$ 為orthogonal。

(5) $u(t)=x(t)*h(t)$ ， $v(t)=y(t)*h(t)$

$h(t)$: LTI channel之impulse response.

$$\int_{-\infty}^{\infty}u(t)v^*(t)dt=\int_{-\infty}^{\infty}U(f)V^*(f)df=\int_{-\infty}^{\infty}X(f)H(f)Y^*(f)H^*(f)df$$

$$=\int_{-\infty}^{\infty}X(f)Y^*(f)\left|H(f)\right|^2df=\int_{-\infty}^{\infty}0\cdot\left|H(f)\right|^2df=0$$

對任何 $h(t)$ 而言，$u(t)$ 與 $v(t)$ 均為orthogonal。

試題30

Shown below are the characteristics of two low-pass filters $H_1(f)$ and $H_2(f)$ and two high-pass filter $H_3(f)$ and $H_4(f)$. In the following