2. 已知已滑動 $\Rightarrow \begin{cases} (1)\ f=\mu_k N \\ (2)\ a_G \neq r\alpha \end{cases}$

a_G 雖然變爲未知數，但 $f=\mu_k N$ 變爲已知數，故總未知數不變。

3. 已知 μ 值 $\Rightarrow$ 摩擦狀況未明 $\Rightarrow \begin{cases} (1)\text{假設無滑動。} \\ (2)\text{檢核。} \end{cases}$

4. 純滾動⇒圓心作直線運動，

$V_G=r\omega$ （$\rightarrow$）

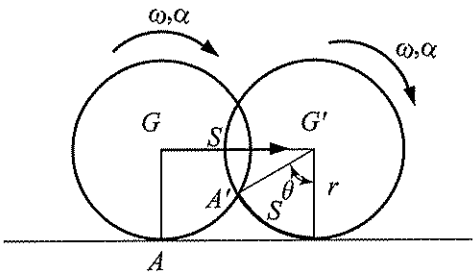
$a_G=r\alpha$ （$\rightarrow$）

圓盤上其它點繞圓心作圓周運動。故圓盤上其它點之速度及加速度要由圓心去推。

$s=r\theta$ （直線）

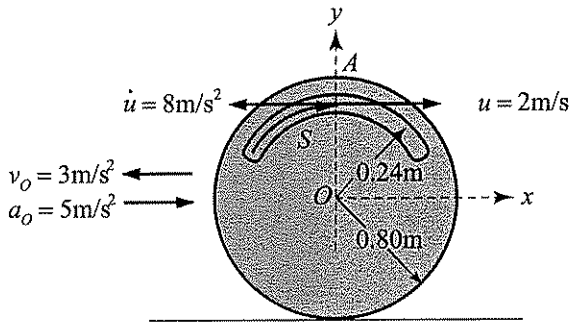
$\dot{s}=v=r\dot{\theta}=r\omega$

$\ddot{s}=a=r\ddot{\theta}=r\alpha$



範例 1

The disk rolls without slipping on the horizontal surface, and at the instant represented, the center O has the velocity and acceleration shown in the following figure. For this instant, the particle A has the indicated speed $\mu=\dot{s}$ and time-rate-of-change of speed $\dot{\mu}=\ddot{s}$, both relative to the disk. Determine the absolute velocity and acceleration of particle A.



（台科機械）

【解】

(1)將動座標 xyz 附在 O 點上。

$\because \vec{v}_O = -3\vec{i}$ (m/s)

$\vec{a}_O = 5\vec{i}$ (m/s^2)

$\therefore \vec{\omega}_{xyz} = \dfrac{3}{0.8}\vec{k} = 3.75\vec{k}$ (rad/s)

$\vec{\alpha}_{xyz} = \dfrac{5}{0.8}\vec{k} = -6.25\vec{k}$ (rad/s^2)

$(\vec{v}_A)_{Oxyz} = 2\vec{i}$ (m/s)

$(\vec{a}_A)_{Oxyz} = -8\vec{i} - \dfrac{2^2}{0.24}\vec{j} = -8\vec{i} - 16.67\vec{j}$ (m/s^2)

(2) $\vec{v}_A = \vec{v}_O + \vec{\omega}_{xyz} \times \vec{r}_{A/O} + (\vec{v}_A)_{Oxyz} = -3\vec{i} + 3.75\vec{k} \times 0.24\vec{j} + 2\vec{i} = -1.9\vec{i}$ (m/s)

$\vec{a}_A = \vec{a}_O + \vec{\alpha}_{xyz} \times \vec{r}_{A/O} + \vec{\omega}_{xyz} \times (\vec{\omega}_{xyz} \times \vec{r}_{A/O}) + 2\vec{\omega}_{xyz} \times (\vec{v}_A)_{Oxyz} + (\vec{a}_A)_{Oxyz}$

$= 5\vec{i} + (-6.25\vec{k}) \times 0.24\vec{j} + 3.75\vec{k} \times (3.75\vec{k} \times 0.24\vec{j})$

$+ 2(3.75\vec{k} \times 2\vec{i}) + (-8\vec{i} - 16.67\vec{j})$

$= -1.5\vec{i} - 5.042\vec{j}$ (m/s^2)

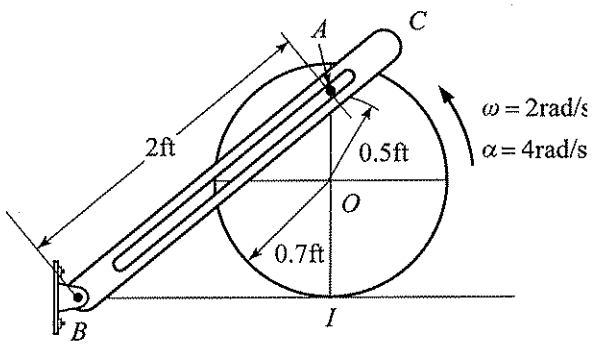
範例 2

At a given instant, the disk rolls without slipping such that it has an angular velocity $\omega = 2$rad/s and angular acceleration $\alpha = 4$rad/s^2. The peg at A is fixed to the disk and located above the disk center O with $OA = 0.5$ft. Point I lies on the periphery of the disk of radius 0.7ft and contacts with a smooth plane. At this instant, determine

(1)the velocity of point A and the angular velocity of the slotted link BC.

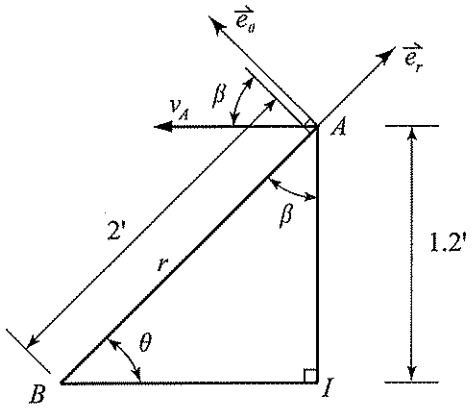
(2)the acceleration of point I and point A.

(3)the angular acceleration of the slotted link BC.



【解】

(1)



①因圓盤純滾動，故 $\vec{v}_I = 0$

$\therefore \vec{v}_A = 1.2 \times 2 = 2.4\,(\text{ft/s})(\leftarrow)$

②利用極座標

$\because r = 2\,(\text{ft})$

$$\theta = \sin^{-1}\left(\frac{1.2}{2}\right) = 36.9^\circ$$

$\therefore \beta = 53.1^\circ$

$$\vec{v}_A = -2.4 \times \sin 53.1^\circ \vec{e}_r + 2.4 \times \cos 53.1^\circ \vec{e}_\theta$$

$$= -1.919\vec{e}_r + 1.441\vec{e}_\theta\,(\text{ft/s}) = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta$$

$$\dot{r} = -1.919\,(\text{ft/s})$$

$$(2)(\dot{\theta}) = 1.441$$

$$\dot{\theta} = \omega_{BC} = 0.721\,(\text{rad/s}) \quad (\curvearrowleft)$$

(2)因圓盤是作純滾動，而純滾動的運動行爲是圓心作直線運動，其它點繞圓心作圓周運動，故求 $\vec{a}_A$ 及 $\vec{a}_I$ 要先求 $\vec{a}_O$ ，再代入剛體運動學公式中求 $\vec{a}_A$ 及 $\vec{a}_I$ 。

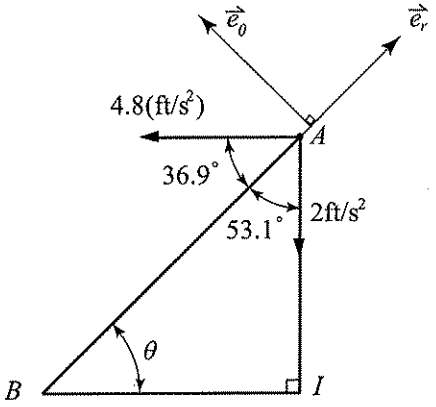
$\therefore \vec{a}_O = 0.7\alpha = 0.7 \times 4 = 2.8\,(\text{ft/s}^2)(\leftarrow)$

$$\vec{a}_A = \vec{a}_O + \vec{\alpha} \times \vec{r}_{A/O} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{A/O})$$

$$= -2.8\vec{i} + 4\vec{k} \times (0.5\vec{j}) - (2)^2(0.5\vec{j}) = -4.8\vec{i} - 2\vec{j}\,(\text{ft/s}^2)$$

$$\therefore \vec{a}_I = \vec{a}_O + \vec{\alpha}\times\vec{r}_{I/O} + \vec{\omega}\times(\vec{\omega}\times\vec{r}_{I/O}) = -2.8\vec{i} + 4\vec{k}\times(-0.7\vec{j}) - (2)^2(-0.7\vec{j})$$
$$= 2.8\vec{j}\,(\text{ft/s}^2)$$

⑶利用極座標分析：



$$\therefore \vec{a}_A = 4.8(-\cos 36.9°\vec{e}_r + \sin 36.9°\vec{e}_\theta) + 2(-\cos 53.1°\vec{e}_r - \sin 53.1°\vec{e}_\theta)$$
$$= -5.04\vec{e}_r + 1.283\vec{e}_\theta\,(\text{ft/s}^2) = (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta$$
$$(2)(\ddot{\theta}) + 2(-1.919)(0.721) = 1.283$$
$$\ddot{\theta} = \alpha_{BC} = 2.025\,(\text{rad/s}^2)\ (\circlearrowleft)$$

範例 3

As shown in Figure, a disk rolls without slipping such that it has an angular acceleration of $\alpha = 4\text{rad/s}^2$ and angular velocity of $\omega = 2\text{rad/s}$ at the instance shown. Determine the accelerations of point A and B on the link and the link's angular acceleration. （成大機械）