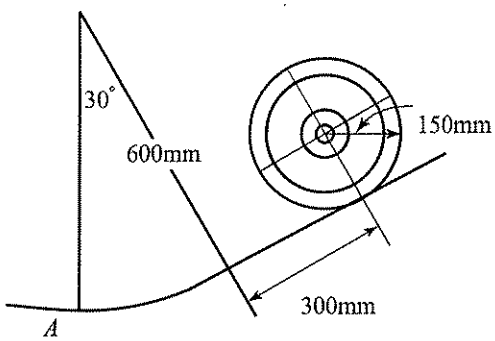
$$\Rightarrow \begin{cases} \alpha = 13.8(\text{rad/s}^2) \\ N = 91.3(\text{N}) \\ f = 20.1(\text{N}) \end{cases}$$

範例 11

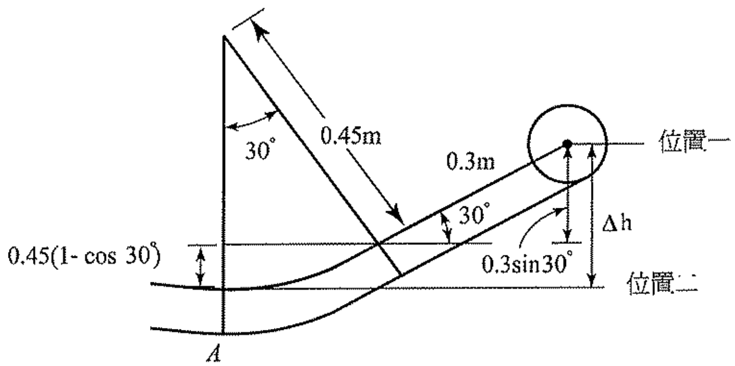
The center of the 100kg wheel with centroidal radius of gyration of 100mm has a velocity of 0.6m/s down the incline in the position shown. Calculate the normal reaction N under the wheel as rolls past position A. Assume that slipping occurs.



（台科機械）

【解】

(1)



$$\because\ \omega_1 = \frac{0.6}{0.15} = 4(\text{m/s})$$

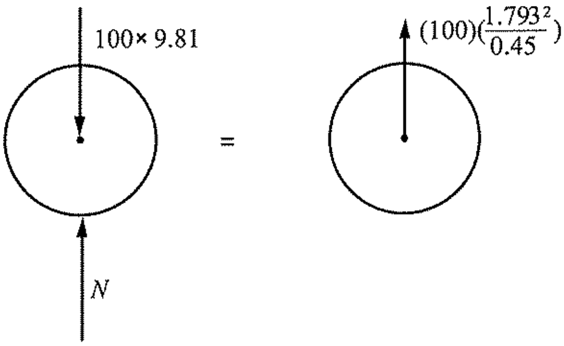
$$\omega_A = \frac{v_A}{0.15} = 6.667 v_A\,(\text{m/s})$$

$\therefore$ 由功能原理：$\Rightarrow T_1 + W_{1\to 2} = T_2$

$$\Rightarrow \frac{1}{2}\left[(100)(0.1)^2 + (100)(0.15)^2\right](4)^2 + (100\times 9.81)[0.3\times\sin 30° + 0.45(1-\cos 30°)]$$

$$= \frac{1}{2}\left[(100)(0.1)^2 + (100)(0.15)^2\right](6.667v_A)^2 \Rightarrow v_A = 1.793\,(\text{m/s})$$

(2)曲率半徑用 0.45 (m)

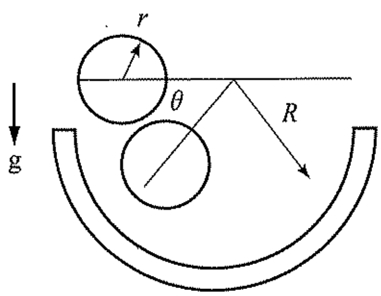


$\because \sum F_y = ma_y$ （↑+）

$$\therefore N - 100\times 9.81 = (100)\left(\frac{1.793^2}{0.45}\right) \Rightarrow N = 1695.4\,(\text{N})(\uparrow)$$

範例 12

The Lord of the Rings is the greatest adventure ever told. Five wizards were sent to Middle-earth to serve as advisors to oppose evil. Wizards possess great powers of body and mind. One day, a wizard released the ring of mass m and radius r from rest on the edge of a large hemispherical bowl of radius R. The ring rolled without slipping during the motion. Determine:

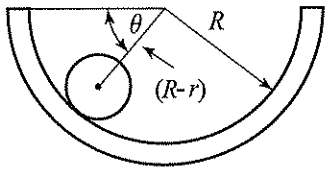


(1)the maximum angular velocity of the ring.

(2)the force N exerted by the path on the ring as a function of θ, and

(3)the rolling friction force as a function of θ. （台大機械）

【解】



(1)①由功能原理求 θ 處之 $\omega=$？

$$\Rightarrow T_1+W_{1\to 2}=T_2$$

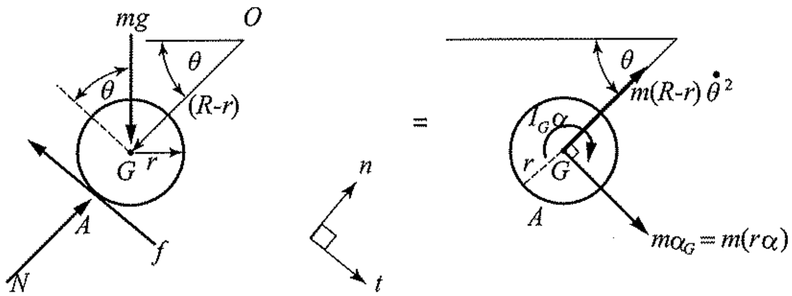
$$\Rightarrow 0+mg(R-r)\sin\theta=\frac{1}{2}(mr^2+mr^2)\omega^2$$

$$\therefore\ \omega=\frac{\sqrt{g(R-r)\sin\theta}}{r}$$

② $\theta=\dfrac{\pi}{2}$ 時 ω 有極大值：

$$\therefore\ \omega_{\max}=\frac{\sqrt{g(R-r)}}{r}$$

(2)求 θ 處時之 N 及 f，以 θ 來表示：



①求ring本身運動時之 α 與ring繞著 O 點旋轉之 $\ddot{\theta}$ 之關係：

$$\because\ a_G=r\alpha=(R-r)\ddot{\theta}$$

$$\therefore\ \alpha=\frac{(R-r)}{r}\ddot{\theta}$$

② $\because\ \sum M_A=(\sum M_A)_{eff}$ （$\circlearrowright +$）

$$\Rightarrow (mg\cos\theta)(r)=(mr^2)(\alpha)+ma_G(r)=(2mr^2)\alpha$$

$$\Rightarrow (mg\cos\theta)(r)=(2mr^2)\left(\frac{(R-r)\ddot{\theta}}{r}\right)\Rightarrow m(R-r)\ddot{\theta}=\frac{1}{2}mg\cos\theta \cdots\cdots\text{(A)}$$

$$\because\ \sum F_t=ma_t \quad (\searrow +)$$

$$\therefore\ mg\cos\theta-f=m(R-r)\ddot{\theta} \cdots\cdots\text{(B)}$$

將(A)式代入(B)式中得：$\Rightarrow f=\dfrac{1}{2}mg\cos\theta$

③求ring本身之 ω 與繞 O 點旋轉之 $\dot{\theta}$ 之關係：

$\because\ v_G = r\omega = (R-r)\dot{\theta}$

$\therefore\ \dot{\theta} = \dfrac{r}{(R-r)}\omega = \left[\dfrac{r}{(R-r)}\right]\left[\dfrac{\sqrt{g(R-r)\sin\theta}}{r}\right] = \dfrac{\sqrt{g(R-r)\sin\theta}}{(R-r)}$ ……(C)

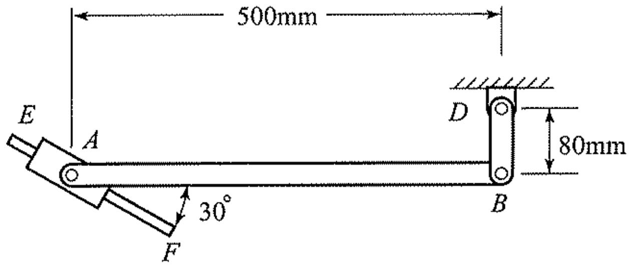
④ $\because\ \sum F_n = ma_n$ （↗＋）

$\therefore\ -mg\sin\theta + N = m(R-r)\dot{\theta}^2$ ……(D)

將(C)式代入(D)式中得：$\Rightarrow N = 2mg\sin\theta$

範例⑬

The 3kg uniform rod AB is connected to crank BD and to a collar of negligible mass, which can slide freely along rod EF. Knowing that in the position shown crank BD rotates with an angular velocity of 15rad/s and an angular acceleration of 60rad/s^2, both clockwise, determine the reaction at A.



（成大機械）

【解】

(1) $\because\ \vec{v}_B = (-15\vec{k})\times(-0.08\vec{j}) = -1.2\vec{i}$ (m/s)

$\vec{a}_B = (-60\vec{k})\times(-0.08\vec{j}) - (15)^2(-0.08\vec{j}) = -4.8\vec{i} + 18\vec{j}$ (m/s^2)

假設 $\vec{v}_A = v_A$ （$\searrow$ 30°）

$\therefore\ \vec{v}_A = v_A\cos 30^\circ\vec{i} - v_A\sin 30^\circ\vec{j} = 0.866v_A\vec{i} - 0.5v_A\vec{j}$ ，假設 $\vec{\omega}_{AB} = \omega_{AB}\vec{k}$

又：$\vec{v}_B = \vec{v}_A + \vec{\omega}_{AB}\times\vec{r}_{B/A} = 0.866v_A\vec{i} - 0.5v_A\vec{j} + \omega_{AB}\vec{k}\times(0.5\vec{i})$

$\Rightarrow \begin{cases} -1.2 = 0.866v_A \\ 0 = -0.5v_A + 0.5\omega_{AB} \end{cases} \Rightarrow v_A = -1.386\text{(m/s)}(\nwarrow)$

$\omega_{AB} = -1.386$ (rad/s) （↻）

(2)假設 $\vec{a}_A = a_A$ （$\searrow$ 30°）

$\therefore\ \vec{a}_A = 0.866a_A\vec{i} - 0.5a_A\vec{j}$ ，假設 $\vec{\alpha}_{AB} = \alpha_{AB}\vec{k}$

$\because\ \vec{a}_B = \vec{a}_A + \vec{\alpha}_{AB}\times\vec{r}_{B/A} + \vec{\omega}_{AB}\times(\vec{\omega}_{AB}\times\vec{r}_{B/A})$

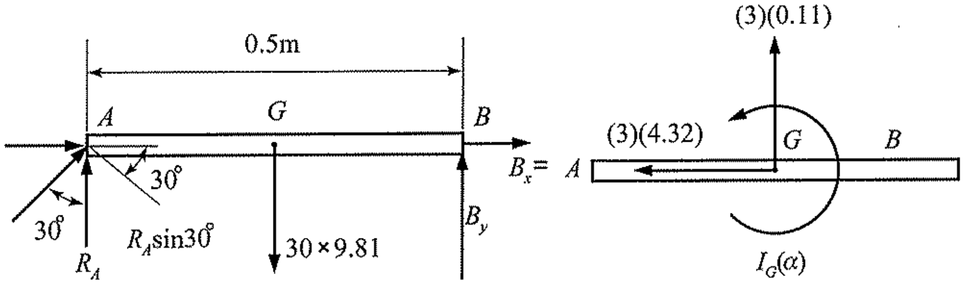
$$\Rightarrow -4.8\vec{i}+18\vec{j}=0.866a_A\vec{i}-0.5a_A\vec{j}+\alpha_{AB}\vec{k}\times(0.5\vec{i})-(1.386)^2(0.5\vec{i})$$

$$\Rightarrow \begin{cases}-4.8=0.866a_A-0.96\\ 18=-0.5a_A+0.5\alpha_{AB}\end{cases}\Rightarrow\begin{cases}a_A=-4.434\,(\text{m/s}^2)(\nwarrow)\\ \alpha_{AB}=31.57\,(\text{rad/s}^2)(\circlearrowleft)\end{cases}$$

(3)求 $(a_G)_x$，$(a_G)_y \Rightarrow AB$ 桿

$$\vec{a}_G=\vec{a}_B+\vec{\alpha}_{AB}\times\vec{r}_{G/B}+\vec{\omega}_{AB}\times(\vec{\omega}_{AB}\times\vec{r}_{G/B})$$
$$=-4.8\vec{i}+18\vec{j}+(31.57\vec{k})\times(-0.25\vec{i})-(1.386)^2(-0.25\vec{i})$$
$$=-4.32\vec{i}+10.11\vec{j}\,(\text{m/s}^2)$$

(4)取 AB 桿分析：



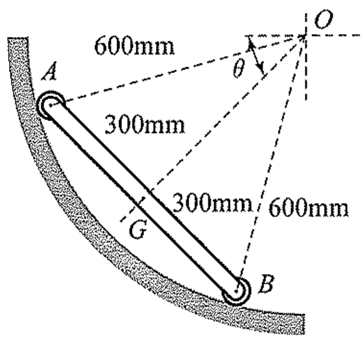
$$\because \sum M_B=(\sum M_B)_{eff}\quad(+\circlearrowleft)$$

$$\Rightarrow -R_A\times\cos30°\times0.5+3\times9.81\times0.25=\left(\frac{1}{12}(3)(0.5)^2\right)(31.57)-(3)(0.11)\times0.25$$

$$\Rightarrow R_A=29.95\,(\text{N})(\nearrow)$$

範例 14

The 24kg uniform slender bar AB is mounted on end rollers of negligible mass and rotates about the fixed point O as it follows the circular path in the vertical plane. The bar is released from a position which gives it an angular velocity $\omega=2\text{rad/sec}$ as it passes the position $\theta=45°$. Calculate the forces F_A and F_B exerted by the guide on the rollers for this instant.



（台科）