➠答題參考

1. 斷面極慣性矩計算

$$J_a = \frac{\pi}{2} r_a^4 \text{ ， } J_b = \frac{\pi}{2}(r_b^4 - r_a^4)$$

2. A、B兩材料各自的最大剪應力計算

設 T_B 爲作用在Hollow Tube上的扭矩；T_A 爲作用在Solid Core上的扭矩。

則

$$T_B + T_A = T$$

同一個複合斷面因材料間緊密接合故具有相同的扭轉角（扭率）。

$$\phi_A = \frac{T_A L}{G_a J_a}$$

$$\phi_B = \frac{T_B L}{G_b J_b}$$

$$\phi_A = \phi_B \Rightarrow T_B = T_A \frac{G_b J_b}{G_a J_a}$$

$$T_B + T_A = T = T_A\left(\frac{G_b J_b}{G_a J_a} + 1\right) \Rightarrow T_A = \frac{G_a J_a}{G_a J_a + G_b J_b} T$$

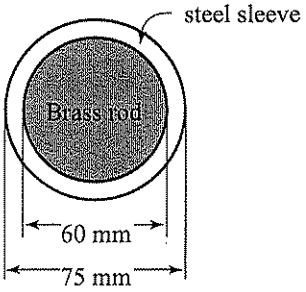
$$T_B = \frac{G_b J_b}{G_a J_a + G_b J_a} T$$

$$(\tau_a)_{\max} = \frac{T_A r_a}{J_a} = (r_a)\left(\frac{G_a}{G_a J_a + G_b J_b}\right) T$$

$$(\tau_b)_{\max} = \frac{T_B r_b}{J_b} = (r_b)\left(\frac{G_b}{G_a J_a + G_b J_b}\right) T$$

試題 ★★★

A composite shaft is formed of two materials, an outer sleeve of steel ($G_s = 80$ GPa) and an inner rod of brass ($G_b = 36$ GPa), as shown in Figure. The outer diameters of the two parts are 75mm and 60mm. Assume that the allowale shear stresses are $\tau_s = 65$ MPa and $\tau_b = 25$ MPa in the steel and brass, respectively.



1. What is the ration of the shear stress at the outer diameter of the steel sleeve to the shear stress at the outer diameter of brass rod?
2. Which part reaches allowable shear stress first?
3. What is the maximum permissible torque T that may be applied to the shaft?

（94台大應力第二題、95地方特考三等、97中興機械第一題類題、98清大動機第三題）

➠破題而入

1. 複合材料緊密接合。

2. 銅鋼間在交接處有相同應變而非相互分離。注意，一個斷面只有一個扭轉角（扭率），所以銅鋼因緊密接合而導致的複合材料桿變形諧和條件爲 $\theta_b = \theta_s = \theta$ 。

➠答題參考

1. 計算剪應力比【觀念25、18】

依【觀念25】複合材料緊密接合變形諧和條件

$\theta_b = \theta_s = \theta$

依【觀念18】之(3.3)圓斷面扭轉剪應力公式

$$\tau_b = G_b\theta_b\rho_b = (36)(30)\theta \quad ; \quad \frac{\tau_s}{\tau_b} = \frac{(80)(37.5)\theta}{(36)(30)\theta} = 2.778$$
$$\tau_s = G_s\theta_s\rho_s = (80)(37.5)\theta$$

2. 判斷銅、鋼誰先降伏？【觀念18】

依題示條件，計算（作用應力）與（容許強度）比值，比值較大者表示先用完他的容許強度空間，先達到容許強度值。

$$\frac{\tau_s}{(\tau_s)_{\text{allow}}} : \frac{\tau_b}{(\tau_b)_{\text{allow}}} = \frac{2.778}{65} : \frac{1}{25} = 1.068:1 \text{ 表示鋼先降伏}$$

而鋼套降伏時，銅的最大剪應力爲

$$\tau_b = \frac{(\tau_s)_{\text{allow}}}{2.778} = \frac{65}{2.778} = 23.4 \text{ MPa}$$

3. 計算最大容許外加扭矩【觀念18】

依【觀念18】之(3.3)圓斷面扭轉剪應力公式

$$T_s = \frac{(65)\left[\dfrac{(\pi)(75^4 - 60^4)}{32}\right]}{37.5} = 3178874.523 \text{ N-mm}$$

$$T_b = \frac{(23.4)\left[\frac{(\pi)(60^4)}{32}\right]}{30} = 992429.119 \text{ N-mm}$$

$$T = T_s + T_b = 4171303.642\text{N-mm} = 4.171\text{kN-m}$$

《題型 3-9》扭轉組合誤差

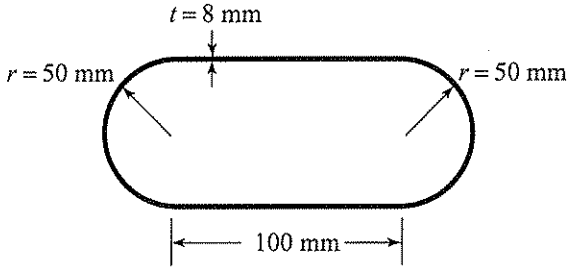
近年來此題型雖尚未有相關試題出現，但同學仍需密切注意！

《題型 3-10》封閉薄壁管

試題 ★★

Calculate the shear stress τ and the angle of twist ϕ for a steel thin-walled tube ($G = 76\text{GPa}$) having the cross section shown in the Figure. The tube length $L = 1.5\text{m}$ and is subjected to a torque $T = 15\text{kNm}$. (Noting that the radius r is measured from the center to the median line of the wall of the tube)

（91機械高考、96交大機械第三題）



破題而入

thin-walled tube：薄壁管。$\tau = \dfrac{T}{2A_m t}$ 、$\phi = \dfrac{TL}{GJ_c}$ 。

答題參考

1. 斷面性質計算

$$A_m = \pi r^2 + 2br = (\pi)(50)^2 + (2)(100)(50) = 17850\text{mm}^2$$

$$L_m = 2b + 2\pi r = (2)(100) + (2)(\pi)(50) = 514.2\text{mm}$$

$$J_C=\frac{4A_m^2}{\oint\frac{dS}{t}}=\frac{4A_m^2t}{L_m}=\frac{(4)(8)(17850)^2}{514.2}=19.83\times10^6\,\text{mm}^4$$

2. 薄壁管扭轉剪應力計算【觀念24】

$$\tau=\frac{T}{2A_mt}=\frac{(15\times1000^2)}{2(8)(17850)}=52.52\text{MPa}$$

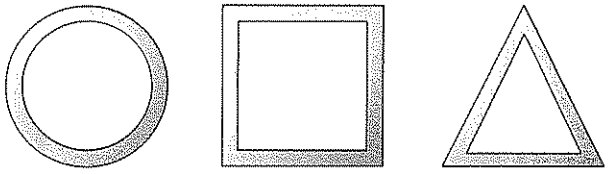
3. 薄壁管扭轉角計算【觀念24】

$$\phi=\frac{TL}{GJ_C}=\frac{(15\times1000)(1.5\times1000)}{(76)(19.83\times10^6)}=0.01493\text{rad}=0.855°$$

試題 ★★★

用同樣材料、同樣厚度的板製成封閉薄壁空心軸。考慮下圖所示三種不同形狀的截面：圓形、正方形、正三角形。如果三種形狀空心軸薄壁中心線所包含的截面積相同，能容許的最大剪應力相同，請問那一種截面可以承受最大的扭矩？

(A)圓形　(B)正方形　(C)正三角形　(D)可承受最大扭矩相同　(E)不一定和軸的長度有關　（95台大機械第十題）



破題而入

1. 厚度，薄壁中心線所包含截面積、容許剪應力皆相同。

2. 三截面t、A_m、τ_{allow}相等。

答題參考

◎求三截面最大扭矩【觀念24】

依【觀念24】之(3.10)薄壁剪應力公式$\tau=\dfrac{T}{2A_mt}$可知

$$T_{\max}=(2)(\tau_{\text{allow}})(A_m)(t)$$

因爲三截面容許剪應力、同厚度、薄壁中心線所包含截面積皆相同故τ_{allow}、t、A_m亦相同。所以$T_{\max}$也相同。