試題 ★★★

A cube of cast iron with sides of length 75 mm is tested in a laboratory under triaxial stress. Gages mounted on the specimen show that the compressive strains are $\varepsilon_x = -350\times10^{-6}$ and $\varepsilon_y = \varepsilon_z = -65\times10^{-6}$.

Please determine the following quantities: (1) the normal stress σ_x, σ_y, σ_z acting on the x, y, and z faces of the cube. (2) The maximum shear stress in the material. (Assume $E = 96$ GPa $\nu = 0.25$) （98交大機械第一題）

破題而入

1. triaxial stresses三軸應力：三個方向皆受到正向應力作用。

2. 最大剪應力：因爲是三軸應力，所以應該計算絕對最大剪應力 $(\tau_{\max})_{abs}$。

答題參考

1. 計算三軸應力

依三軸廣義虎克定律

$$\varepsilon_x = -350\times10^{-6} = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) \cdots\cdots ①$$

$$\varepsilon_y = -65\times10^{-6} = \frac{1}{E}(\sigma_y - \nu\sigma_x - \nu\sigma_z) \cdots\cdots ②$$

$$\varepsilon_z = -65\times10^{-6} = \frac{1}{E}(\sigma_z - \nu\sigma_y - \nu\sigma_x) \cdots\cdots ③$$

由② − ③

$0 = (1+\nu)(\sigma_y - \sigma_z) \Rightarrow \sigma_y = \sigma_z$ 代入①

① × ν + ②

$$-350\times10^{-6}\nu - 65\times10^{-6} = \frac{\sigma_y}{E}(1-\nu-2\nu^2)$$

$$\Rightarrow -152.510^{-6} = \frac{\sigma_y}{96\times1000}(0.625)$$

$\Rightarrow \sigma_y = -23.424\text{MPa} = \sigma_z$（壓應力）

$\sigma_x = (-350\times10^{-6})(96000) + (2)(0.25)(-23.424) = -45.312\text{MPa}$（壓應力）

2. 計算 $(\tau_{\max})_{abs}$

因爲x、y、z方向皆無剪應力，所以σ_x、σ_y、σ_z都是主應力σ_1、σ_2、

σ_3

$$(\tau_{\max})_{\text{abs}} = \max\left\{\left|\frac{\sigma_1-\sigma_2}{2}\right|,\left|\frac{\sigma_2-\sigma_3}{2}\right|,\left|\frac{\sigma_1-\sigma_3}{2}\right|\right\}$$

$$= \max\left\{\left|\frac{-45.312+23.424}{2}\right|,\left|\frac{-23.424+23.424}{2}\right|,\left|\frac{-45.312+23.424}{2}\right|\right\}$$

$$= 10.944\text{MPa}$$

試題 ★★★

An aluminum alloy plate ($E = 70\text{GPa}$, $\nu = 0.33$) of dimensions $a = 300$ mm, $b = 400$ mm, and thickness $t = 10$ mm is subjected to biaxial stresses as shown in Figure P2. Calculate the change in *1.* the length of AB, *2.* the volume of the plate. （95交大機械第二題、96中興機械第四題）

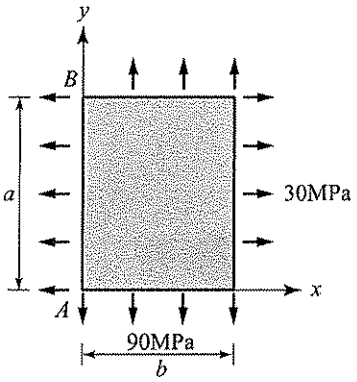

Figure P2

破題而入

1. AB向就是 y 向所以要找 ε_y ： $\delta_{AB} = (\varepsilon_y)(a)$ 。

2. 是平面應力態喔！注意 z 向應變不爲零！

3. 體積改變量 ΔV 與三向應變有關： $(\varepsilon_V)(V) \cong (\varepsilon_x + \varepsilon_y + \varepsilon_z)(V)$ 。

答題參考

1. 求AB長度改變量【觀念51、50】

依【觀念51】之(6.24) y 向應力應變關係與【觀念50】之(6.19) y 向尺寸變化量計算式