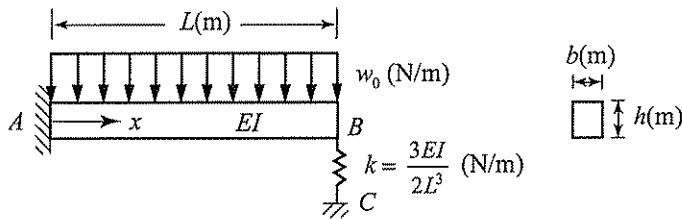


• 試題 •

★★★

A beam, having length L (m) with a rectangular cross section of height h and width b and subjected to a uniformly distributed loading w_0 (N/m), as shown in Fig., is supported by the fixed wall at A and a linearly elastic spring BC at B with spring constant $k = 3EI/2L^3$ (N/m). If the spring is unstretched when the beam is not yet deformed,

1. determine the support reactions at points A and B ;
2. plot the shear force and bending moment diagrams of the beam;
3. calculate the maximum bending normal stress, the maximum shear stress and the maximum deflection;
4. determine the deflection of the beam at point B by Castigliano's second theorem.



(95地方特考三等、97成大機械第五題)

⇒ 破題而入

1. 卡二定理遇到具有彈簧樑：設彈簧的內力為贅力。利用 $\frac{\partial U}{\partial (\text{贅力})} = 0$ 反求贅力。
2. 要求 B 點撓度：不用在 B 點設虛力，因為（彈簧縮短量）=（ B 點向下撓度）。
3. 要求最大撓度：因為 B 點可以移動，所以最大撓度有可能出現在 B 點以及樑中轉角為零處。我們應該先使用積分法找出轉角為零處的撓度，並與 B 點撓度取較大值為答。

⇒ 答題參考

1. 卡二定理分析【觀念65】
 - (1)此樑為一度靜不定，設彈簧壓力為贅力。

8-40 材料力學 (Mechanics of Materials)

$$(2) M(x) = \frac{w_0}{2} x^2 - N_s x$$

$$\frac{\partial M(x)}{\partial N_s} = -x ; \quad \frac{\partial N_s}{\partial N_s} = 1$$

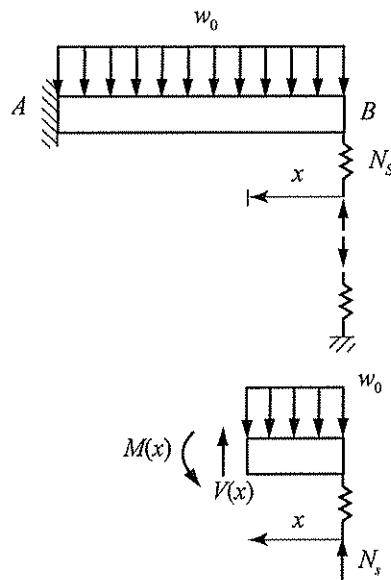
$$(3) \frac{\partial U}{\partial N_s} = 0 = \int_0^L \frac{M(x)}{EI} \left(\frac{\partial M}{\partial N_s} \right) dx + \frac{N_s}{k} \left(\frac{\partial N_s}{\partial N_s} \right)$$

$$= \frac{1}{EI} \int_0^L \left(\frac{-w_0}{2} x^3 + N_s x^2 \right) dx + \frac{N_s}{k}$$

$$= \frac{1}{EI} \left(\frac{-w_0 L^4}{8} + \frac{N_s L^3}{3} \right) + \frac{2 N_s L^3}{3 EI}$$

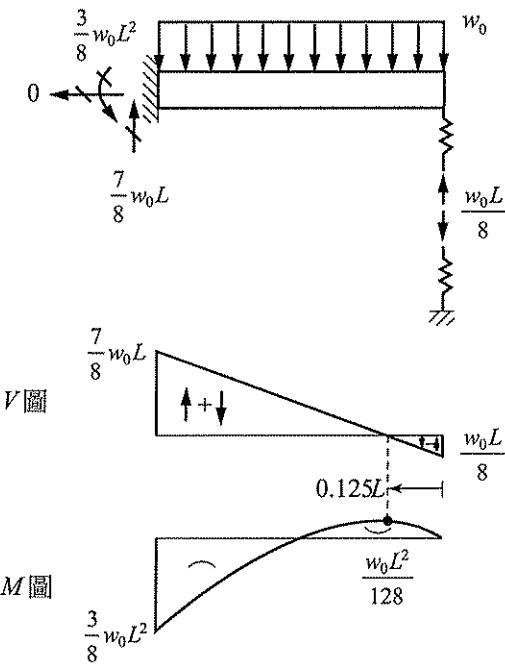
$$\Rightarrow N_s = \frac{w_0 L}{8} \quad (\text{同假設方向，為壓力})$$

$$(\text{B點向下撓度}) = (\text{彈簧縮短量}) \quad \Delta_B = \frac{N_s}{k} = \frac{w_0 L^4}{12 EI} \quad (\downarrow)$$



2. 反力內力分析【觀念01】

已知 $N_s = \frac{w_0 L}{8}$ ，故可由力平衡求出 A、B 點反力及 V 圖、 M 圖



3. 橫剖面應力分析

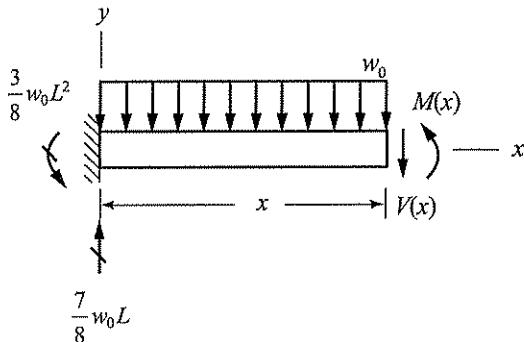
(1) 橫剖面上最大彎曲正向應力

$$\sigma_{\max} = \frac{M_{\max} y_{\max}}{I} = \frac{\left(\frac{3}{8}w_0L^2\right)\left(\frac{h}{2}\right)}{\frac{bh^3}{12}} = \frac{9w_0L^2}{4bh^2} \quad (\text{A點固定端的上、下緣})$$

(2) 橫剖面上最大彎曲剪應力

$$\tau_{\max} = \frac{3V_{\max}}{2A} = \frac{(3)\left(\frac{7}{8}w_0L\right)}{(2)(bh)} = \frac{21w_0L}{16bh} \quad (\text{A點斷面的中間中性軸處})$$

4. 最大撓度計算【觀念40】



採積分法，設定第一象限座標

$$M(x) = \frac{7}{8}w_0Lx - \frac{3}{8}w_0L^2 - \frac{w_0}{2}x^2$$

$$EIv'' = M(x)$$

$$EIv' = \frac{7}{16}w_0Lx^2 - \frac{3}{8}w_0L^2x - \frac{w_0}{6}x^3 + C_1$$

$$EIv = \frac{7}{48}w_0Lx^3 - \frac{3}{16}w_0L^2x^2 - \frac{w_0}{24}x^4 + C_1x + C_2$$

$$v(0) = 0 \Rightarrow C_2 = 0$$

$$v'(0) = 0 \Rightarrow C_1 = 0$$

當 $v' = \theta = 0$ 時有 v_{\max}

$$0 = \frac{7}{16}w_0Lx^2 - \frac{3}{8}w_0L^2x - \frac{w_0}{6}x^3$$

$$\Rightarrow x_1 = 0 ; x_2 = 1.3125 + 0.7i ; x_3 = 1.3125 - 0.7i \text{ (皆不合)}$$

故 v_{\max} 出現在 B 點

$$v_{\max} = v(L) = \frac{-w_0L^4}{12EI} \quad (\downarrow)$$

• 試題 •



A linear distributed load w is supported by a cantilever beam and a linear spring of stiffness k (as shown in Figure). The Young's modulus of the beam is E and the moment of inertia of the beam is I . Assume the deflection of the beam is Δ .

1. Determine the shear $V(x)$ and bending moment $M(x)$ of the beam by L ,