$F_s = F_c = 36028\ \ell b$

所以螺栓之應力 σ_s 及鋼管之應力 σ_c 分別為

$$\sigma_s = \frac{F_s}{A_s} = \frac{36028}{\frac{1}{2}}$$

$$= 72.056 \text{ ksi}$$

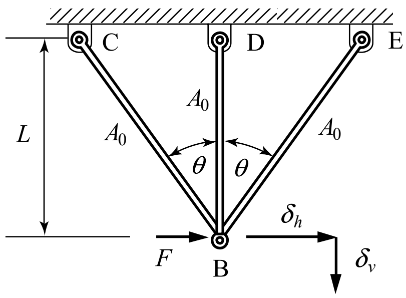
$$\sigma_c = \frac{F_c}{A_c} = \frac{36028}{\frac{3}{4}}$$

$$= 48.037 \text{ ksi}$$

題型4-8 左右不對稱之一般靜不定結構

範題 13

Consider the pin-connected truss as shown in Fig. The three bars have the same cross-sectional area A_0 and Young's modulus E. A horizontal force F is applied to joint B.



(1) Is the structure statically determinate or statically indeterminate?

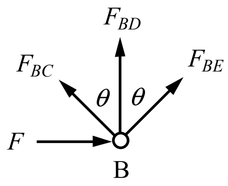
(2) Draw the free-body diagram of joint B and write down the equilibrium equations.

(3) Let δ_h and δ_v indicate the horizontal and vertical displacements of joint B, respectively. Express the change in length of each bar in terms of δ_h and δ_v.

(4) If the three bars have the same allowable load F_{allow}, determine the largest value of the horizontal force F. (90台大機械)

【解析】

⑴因為有三根桿件，故有三個未知軸力，若取點B為分離體，則只能列出兩個獨立方程式，所以結構為一維靜不定。



⑵分別考慮水平方向及垂直方向之靜力平衡，可列出

$$+F-F_{BC}\sin\theta+F_{BE}\sin\theta=0 \quad\cdots\cdots ①$$

$$+F_{BC}\cos\theta+F_{BD}+F_{BE}\cos\theta=0 \quad\cdots\cdots ②$$

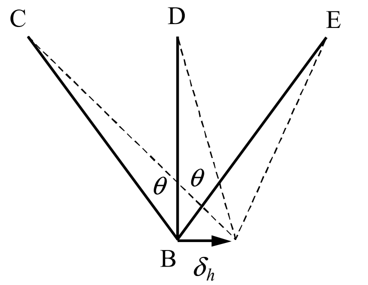
式中F_{BC}、F_{BD}及F_{BE}分別為桿BC、BD及BE之受力。

⑶點B水平變位δ_h引起各桿之變形量如下：

$$\delta_{BC}=\delta_h\sin\theta$$

$$\delta_{BD}=0$$

$$\delta_{BE}=-\delta_h\sin\theta$$



點B垂直變位δ_v引起各桿之變形量如下：

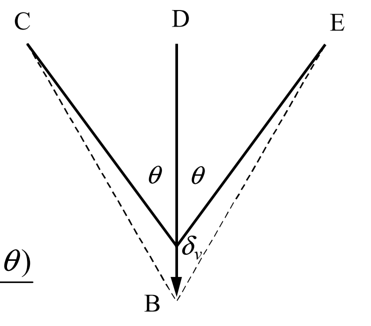
$$\delta_{BC}=\delta_v\cos\theta$$

$$\delta_{BD}=\delta_v$$

$$\delta_{BE}=\delta_v\cos\theta$$

所以點B水平變位δ_h與垂直變位δ_v所引起之各桿之變形量如下：

$$\begin{cases}\delta_{BC}=+\delta_h\sin\theta+\delta_v\cos\theta\\ \delta_{BD}=0+\delta_v=\delta_v\\ \delta_{BE}=-\delta_h\sin\theta+\delta_v\cos\theta\end{cases} \quad\cdots\cdots ③$$



⑷由式③及變形量公式可得到

$$\delta_{BC}=+\delta_h\sin\theta+\delta_v\cos\theta=\frac{F_{BC}(L/\cos\theta)}{A_0E}$$

$$\Rightarrow F_{BC}=\frac{A_0E}{L}(\delta_h\sin\theta\cos\theta+\delta_v\cos^2\theta) \quad\cdots\cdots ④$$

$$\delta_{BD}=0+\delta_v=\delta_v=\frac{F_{BD}L}{A_0E}$$

$$\Rightarrow F_{BD}=\frac{\delta_v A_0 E}{L} \cdots\cdots ⑤$$

$$\delta_{BE}=-\delta_h\sin\theta+\delta_v\cos\theta$$

$$=\frac{F_{BE}(L/\cos\theta)}{A_0E}$$

$$\Rightarrow F_{BE}=\frac{A_0E}{L}(-\delta_h\sin\theta\cos\theta+\delta_v\cos^2\theta) \cdots\cdots ⑥$$

將式④、⑤及⑥代入式①及②，可得到

$$+F-\frac{A_0E}{L}(\delta_h\sin\theta\cos\theta+\delta_v\cos^2\theta)\sin\theta$$

$$+\frac{A_0E}{L}(-\delta_h\sin\theta\cos\theta+\delta_v\cos^2\theta)\sin\theta=0 \cdots\cdots ⑦$$

$$\frac{A_0E}{L}(\delta_h\sin\theta\cos\theta+\delta_v\cos^2\theta)\cos\theta+\frac{\delta_v A_0E}{L}$$

$$+\frac{A_0E}{L}(-\delta_h\sin\theta\cos\theta+\delta_v\cos^2\theta)\cos\theta=0 \cdots\cdots ⑧$$

由式⑧可求得$\delta_v=0$，代入式⑦中，可解出δ_h為

$$\delta_h=\frac{FL}{2A_0E\sin^2\theta\cos\theta} \cdots\cdots ⑨$$

再代入式④、⑤及⑥，即可求出

$$F_{BC}=\frac{F}{2\sin\theta}$$

$$F_{BD}=0$$

$$F_{BE}=-\frac{F}{2\sin\theta}$$

又因為桿件所能承受之容許負載為F_{allow}，故

$$F_{BC}=\frac{F}{2\sin\theta}\le F_{\text{allow}}$$

$$\Rightarrow F\le 2F_{\text{allow}}\sin\theta$$

所以結構所能承受之最大作用力$F=2F_{\text{allow}}\sin\theta$。