入後，C點端將存在反扭矩T_C限制C點端轉回去，故相當於T_C使圓軸轉ϕ角，亦即

$$\frac{T_C \times 5L}{GJ} = \phi = \frac{9t_0 L^2}{2GJ}$$

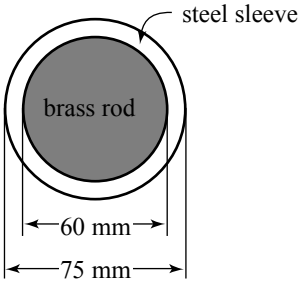
$$\Rightarrow T_C = \frac{9}{10} t_0 L$$

圓軸之殘留應力是因存在反扭矩T_C而引起，而最大殘留剪應力$\tau_{\max}$應發生於圓軸之外表面，其值為

$$\tau_{\max} = \frac{T_C r}{J} = \frac{\frac{9}{10} t_0 L \times \frac{d}{2}}{\frac{\pi \times d^4}{32}} = \frac{72 t_0 L}{5\pi d^3}$$

題型6-8 複合圓軸之靜不定問題

範題 19



A composite shaft is formed of two materials, an outer sleeve of steel ($G_s = 89$ GPa) and an inner rod of brass ($G_b = 36$ GPa), as shown in figure. The outer diameters of the two parts are 75 mm and 60 mm. Assume that the allowable shear stresses are $\tau_s = 65$ MPa and $\tau_b = 25$ MPa in the steel and brass, respectively.

⑴What is the ratio of the shear stress at the outer diameter of the steel sleeve to the shear stress at the outer diameter of brass rod?

⑵Which part reaches allowable shear stress first?

⑶What is the maximum permissible torque T that may be applied to the shaft? （94台大應力(E)）

【解析】

⑴複合圓軸在承受扭矩T時，其剪應變γ仍沿半徑方向呈線性分布，亦即$\gamma=\rho\theta$仍成立，其中ρ為半徑，而θ為單位長度之扭轉角，因此鋼管外徑處之剪應變γ_s與銅桿外徑處之剪應變γ_b之比值應為

$$\frac{\gamma_s}{\gamma_b}=\frac{\frac{75}{2}\times\theta}{\frac{60}{2}\times\theta}=\frac{5}{4}$$

所以鋼管外徑處之剪應力τ_s與銅桿外徑處之剪應力之τ_b之比值為

$$\frac{\tau_s}{\tau_b}=\frac{G_s\gamma_s}{G_b\gamma_b}=\frac{80\times10^9}{36\times10^9}\times\frac{5}{4}=2.78$$

⑵由⑴之結果知鋼管之最大剪應力為銅管最大剪應力之2.78倍，但鋼之容許剪應力τ_s只為銅之容許剪應力τ_b之$65/25=2.6$倍，故可知鋼管會先達到容許剪應力。

⑶鋼管之極慣性矩J_s及銅桿之極慣性矩J_b分別為

$$J_s=\frac{\pi}{32}\times(75^4-60^4)=1.834\times10^6\ \text{mm}^4=1.834\times10^{-6}\ \text{m}^4$$

$$J_b=\frac{\pi}{32}\times60^4=1.272\times10^6\ \text{mm}^4=1.272\times10^{-6}\ \text{m}^4$$

當鋼管外徑處剪應力達到容許剪應力時，其所承受之扭矩T_s可由下式求得：

$$\tau_s=65\times10^6=\frac{T_s\times\frac{75}{2}\times10^{-3}}{1.834\times10^{-6}}$$

$$\Rightarrow T_s=3178.93\ \text{N}\cdot\text{m}$$

而此時銅管外徑處之剪應力τ_b為

$$\frac{\tau_s}{\tau_b}=2.78$$

$$\Rightarrow\tau_b=\frac{\tau_s}{2.78}=23.38\ \text{MPa}$$

故此時圓管所承受之扭矩T_b為

$$\tau_b = 23.38\times10^6 = \frac{T_b \times \frac{60}{2}\times10^{-3}}{1.272\times10^{-6}}$$

$$\Rightarrow T_b = 991.31\ \text{N}\cdot\text{m}$$

所以複合圓軸之最大容許扭矩$T_{\max}$為

$$T_{\max} = T_s + T_b = 3178.93 + 991.31 = 4169.93\ \text{N}\cdot\text{m}$$